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PROCEEDINGS
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A DETERMINATION OF THE DISTANCE TO THE GALACTIC CENTER

By Harlow Shapley

Harvard College Observatory

Communicated February 15, 1939

1. In a southern field of eighty square degrees not far from the galactic center a recently completed study shows six hundred galaxies and more than five hundred variable stars. Of the latter, more than half are of the Cepheid type. We can use the apparent magnitudes of the Cepheids for work on galactic dimensions, and in particular we can make a new estimate of the distance to the nuclear star clouds in Sagittarius.

The field, MWF 269, which has been under study for the past two years, is adjacent to and partly overlapping MWF 233, reported in various earlier contributions. Both fields are of importance in that they help to outline a galactic window—a region relatively near the galactic circle in which the presence of numerous external galaxies argues for little space absorption and for the dependability of the Cepheids in distance measurements.

MWF 269 is centered at $18^h\ 20^m$, $-55°$ (1900), galactic coördinates 307°, $-20°$. The area is that covered by an 8×10-inch plate made with the Metcalf 10-inch triplet at Bloemfontein. On plates with this instrument 301 new variables were found, of which 179 are of the cluster type; also 143 were already known in the area (mostly from the overlapping part of the MWF 233 survey), and of them sixty-seven are of this same useful type. Since the variable star survey with the Metcalf telescope (MF plates) is essentially complete only to magnitude 15.5, we have supplemented it with a series of photographs with the Bruce telescope (A plates) on the central part of the field, increasing the depth by about one magnitude. Once a variable star has been detected on the Bruce plates, it is generally possible to identify and measure it on the MF series. In the study of the cluster variables for the purpose of galactic measurement we have followed the customary plan of measuring only the plates necessary to provide identification of type of variable and a good determination of the median magnitude. The list of new variables in MWF 269 will be published in the

Annals of the Observatory, together with details on the magnitude system, the relative frequency of various types and the distribution of variables over the field.

The frequency of the median magnitudes of the cluster variables found in the MF survey for the entire eighty square degrees of MWF 269 (supplemented with the thirteen previously published cluster variables in this region) is illustrated in figure 1, which shows the number of variables in half magnitude intervals. The maximum at magnitude 15.3 is too near the effective discovery limit of the Metcalf plates to demonstrate certainly a maximum space density at the corresponding distance. It is to test the reality of this maximum, which might be taken to indicate where the line of sight crosses the galactic nucleus, that the frequency distribution of the magnitudes of the variables in a central area of twenty square degrees has been examined on plates taken with the Bruce telescope. These plates carry the essential completeness of the survey to objects with median magnitudes well below 16.0. But to permit the translation of observed apparent magnitudes into distances corrected for space absorption it is necessary first to look at the census of external galaxies in the field.

2. Ten long-exposure A plates with stellar limits near the sixteenth magnitude have been used in a complete survey of MWF 269 for the discovery of galaxies. Their qualities differ considerably, but since there is much overlapping we are confident that the study is complete to the sixteenth magnitude, and that down to magnitude 17.4 the frequency of galaxies can be used satisfactorily to judge the degree of space transparency. To increase the weight of the determination by involving more objects, we have used the galaxies throughout the whole eighty square degrees of MWF 269 in deriving the absorption for the central twenty square degrees. The nebular distribution is fairly uniform over the field, the total numbers in the four quadrants being 149, 166, 174, 179. The nebular magnitude system has been established as for MWF 233.[1] The results from the individual A plates will be published eventually as part of the general survey of galaxies. It is sufficient for the present study to abstract from the ten plates the following magnitude frequency relation, which gives the number of galaxies down to and including various magnitude limits:

To magnitude,	15.0	15.6	16.05	16.5	17.0	17.4
Number,	25	53	82	129	281	542

Following the method outlined in Tercentenary Papers, No. 13,[2] and with N representing the number of galaxies per square degree to the observed magnitude m', we compute the total photographic space absorption from the relation

$$\delta m = m' - m_1 - 1.67 \log N.$$

Again taking $m_1 = 15.4$, we obtain the following results:

m'	15.0	15.6	16.05	16.5	17.0	17.4
N	0.31	0.66	1.02	1.61	3.51	6.78
δm	0.45	0.49	0.63	0.75	0.69	0.61
Weight	1	2	2	2	1	1

The mean value of the absorption, 0.61, is subject, of course, to the assumption that the uniform density hypothesis is useful in this region of the sky, and that the magnitude scale and the adopted space-density parameter, m_1, are reasonably approximate. All of the cluster variables of

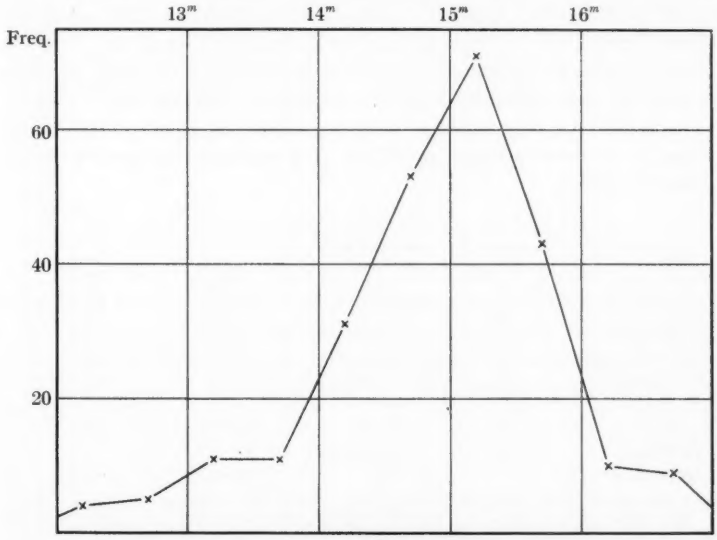

FIGURE 1

Magnitude-frequency diagram for 233 cluster variables in MWF 269. Abscissae are observed median photographic magnitudes; ordinates are numbers of variables in half-magnitude intervals.

MWF 269 lie at distances from the galactic plane, $r \sin \beta$, greater than a thousand parsecs. It is reasonable, therefore, to assume that the space absorption is between the observer and the variable stars (is, in fact, in the neighborhood of the sun), and therefore that $\delta m = m' - m$ can be treated as a zero point correction to the magnitude scale, the same for all the variables whatever their magnitudes and distances.

3. The frequency of the corrected median magnitudes of the cluster variables in the central twenty square degrees of MWF 269, where the survey has been uniformly carried to the workable limit of the Bruce plates, is shown in the second and third columns of table 1. The mean median

magnitude in the third column has been corrected for a space absorption of six-tenths of a magnitude. The corresponding distance is given in the fourth column of the table, computed from the relation $\log r = 0.2\ (m - 10)$, where we express the distance, r, in kiloparsecs and adopt the usual value of the absolute magnitude, $M = 0$.

An inspection of the data in the first three columns of table 1 shows that for the central twenty square degrees, as noted above also for the whole field, there is a maximum frequency at $m' = 15.3$, $m = 14.7$; the corresponding distance is 8.7 kiloparsecs. But the volume in successive magnitude intervals is increasing as r^3, and the space frequency of the variables is falling off with distance because of galactic concentration. The numbers in column 2 must be corrected for both of these factors, and the "cone of observation" thus transformed to an equivalent cylinder in space, parallel to the galactic plane and extending toward galactic longitude 307°. When such a reduction has been made we shall be able to test better the evidence that there is a maximum number of variables per unit volume as we pass the galactic nucleus.

TABLE 1

MAGNITUDE INTERVAL	NUMBER OF VARIABLES	CORRECTED MEAN MAGNITUDE	r (KPC.)	$N_{r,s}$
12.5–12.9	1	12.3	2.88	8.7
13.0–13.4	2	12.55	3.24	13.3
13.5–13.9	2	13.15	4.27	7.1
14.0–14.4	7	13.74	5.60	14.3
14.5–14.9	16	14.22	6.98	22.1
15.0–15.4	27	14.70	8.71	27.0
15.5–15.9	16	15.15	10.72	12.7
16.0–16.4	5	15.60	13.18	3.5
16.5–16.9	1	16.1	16.60	0.7

4. We have as yet little information about the galactic concentration of variables in the vicinity of the galactic nucleus. The best adjustment for the decrease in density in the z coördinate (perpendicular to the galactic plane) that we can now make for the relative numbers in MWF 269 is through use of the recently derived (unpublished) density function for the southern galactic hemisphere. From an investigation of 180 variables in 14 fields covering 1120 square degrees, with galactic latitudes between $-20°$ and $-90°$, we have found

$$N_z = 10^{-0.25(r \sin \beta + 0.4)}. \qquad (1)$$

In deriving this relation a mean total photographic absorption of a quarter of a magnitude was adopted, a value justified by the observed frequency of nebulae in the fields studied.

5. Writing N_r for the observed number of cluster variables in MWF 269 at given intervals along the "cone of observation" (column 2 of table 1).

and N_z, given by equation (1), for the average number of cluster variables in unit volume as a function of distance from the galactic plane, we have for the number in MWF 269 reduced to unit volume at the distance $z = r \sin \beta$,

$$N_{v,z} = k\, N_r / N_z r^3 \qquad (2)$$

where k is a proportionality factor which we shall take as $10^{1.975}$ so that the reduced number of variables $N_{v,z}$ will be equal to the observed number, $N_r = 27$, in the magnitude interval 15.0 to 15.4. This choice of k essentially frees the reduction from concern that the adopted N_z is probably irrelevant in the region of the nucleus; we are, in fact, reducing the numbers in the near and far parts of the cone to the distance $r = 8.7$ kiloparsecs. For MWF 269 the latitude is 20°. Equations (1) and (2) may therefore be written

$$\log N_z = -0.086\, r - 0.10 \qquad (1')$$

and

$$\log N_{v,z} = 2.07 + 0.086\, r + \log N_r - 3 \log r. \qquad (2')$$

The calculated results are in the last two columns of table 1. When the corrected frequencies are plotted against distances it is found that the region of high density is encountered at $r = 8.55$ kpc. If we assume that the cluster variables are concentrated symmetrically toward the center of the nuclear star cloud, in longitude $\lambda = 327°$, we compute the distance to the galactic center from

$$r_c = r \sec\beta \sec(\lambda - 327°)$$

and obtain the value of 9.7 kpc., in general agreement with values recently derived on dynamical grounds and from the space distribution of the globular clusters.

6. There are several factors of uncertainty in the use of the photometric method sketched above which, when summed up, indicate the possibility of a considerable error in the derived result. These uncertainties can be examined by tabulating the estimated mean errors as follows:

in the zero point of the period-luminosity curve	$0^m.15$
in the observed median magnitudes of variables in MWF 269 (including scale and zero point, and observational errors)	0.2
in the space absorption as derived from distribution of galaxies	0.15
in the estimate of the magnitude of maximum frequency	0.1

We conclude therefore that if our basic assumption that the maximum in the magnitude-frequency distribution of cluster variables in MWF 269

implies the more or less symmetrical distribution of variables about the galactic nucleus, the distance to the center of the galaxy is

$$r_c = 9.7 \pm 1.2 \text{ (m. e.)}. \qquad (3)$$

The magnitude frequency curve in the area MWF 233, which is somewhat more distant from the galactic center and in lower latitude, does not yield as clear a maximum as appears in MWF 269. The pronounced maximum found some years ago[3] in MWF 185 is at magnitude 15.75, but the absence of galaxies in this low latitude field near the galactic center does not permit the application of the present method.

A special study is now in progress of the variables in those fields on the border of the galactic nucleus in which nebulae are abundantly found. We should be able with the additional material to reduce the mean error of (3) to half the present value.

[1] *Harv. Circ.*, **411**, 3 (1936).
[2] *Harv. Ann.*, **105**, 243 (1936).
[3] Shapley and Swope, these PROCEEDINGS, **14**, 830 (1928).

SUPERNOVAE AND STELLAR COLLISIONS

BY FRED L. WHIPPLE

HARVARD COLLEGE OBSERVATORY

Communicated February 13, 1939

Introduction.—The hypothesis that a nova may orignate from a collision of two stars would long have been favored except for the theoretical rarity of such collisions. Luyten,[1] Jeans[2] and others have calculated that a stellar collision should occur in our Galaxy only once in 10^{13} to 10^{7} years, whereas several novae occur each year. William Pickering and Nölke[3] suggested that collisions of stars with bodies of asteroidal or planetary dimensions might be sufficient to set off the explosions.

Supernovae represent a catastrophic phenomenon of far greater magnitude and much smaller frequency than do ordinary novae. Modern estimates of the space densities of stars in the nucleus of the galaxy and in external galaxies indicate much greater values than those of a few years ago. It is the purpose of the present study to show that on the basis of recent data the frequency of stellar collisions can be comparable with the frequency of supernovae, and that the collision hypothesis must be considered in a discussion of the origin of supernovae.

1. *Distribution.*—It is necessary first to study the space distribution of supernovae in the external galaxies with which they have been associated.

Table 1 contains, in the eighth column, the apparent distance (r in parsecs) from the nucleus of the galaxy to the supernova for each of the twenty well-authenticated supernovae occurring outside our system. The distance has in each case been corrected (to r_c in the ninth column) for an estimated projection on the line of sight. Each galaxy was observed by the author on photographs, the position of the supernova located and the correction made on the assumption that the supernova lay in the principal plane of the galaxy as estimated from the apparent major and minor axes. The mean ratio of r/r_c for a random distribution of all supernovae directions should be $\pi/4 = 0.79$. The mean ratio from table 1 is 0.82.

TABLE 1

DISTANCES OF SUPERNOVAE FROM NUCLEI

DATE	N. G. C.	TYPE	m_{pg}	$m-M$	$\Delta\alpha \cos\delta$	$\Delta\delta$	r	r_c	REF.
1885	224	S_b	4.5	22.2	− 15″	− 4″	21	34	4
1937	1003	S_c	13.1	26.8	+ 48	− 1	530	740	
1901	2535	SB_c	13.7	28.2	+ 19	+ 7	440	440	5
1920	2608	SB_c	13.6	28.1	− 19	+ 5	400	480	6
1912	2841	S_b	10.6	25.1	− 50	+ 20	270	410	7
1937	4157	S_c	12.0	26.5	+ 42	+ 42	580	930	8
1936	4273	S_c	12.4	26.7	0	+ 29	310	400	9
1926	4303	SB_c	10.4	26.7	− 11	+ 69	740	740	10
1901	4321	S_c	10.5	26.7	−110	+ 4	1170	1170	11
1914	4321	S_c	10.5	26.7	+ 24	−111	1200	1200	11
1895	4424	$S_b$pec	12.5	26.7	+ 75	− 11	810	970	12
1919	4486	E_o	10.1	26.7	− 15	+100	1070	1070	13
1915	4527	S_c	11.3	26.7	+ 44	+ 8	470	620	11
1939	4636	E	10.8	26.7	− 26	+ 20	350	450	
1907	4674	S_b	14.5	29.0	− 10	+ 11	460	680	
1937	I4182	S_c	13.5	24.8	+ 30	+ 40	220	220	
1923	5236	S_c	8.8	24.8	+109	+ 58	550	600	14
1895	5253	Irr.	11.0	25.5	+ 16	+ 23	170	260	15
1934	I4719	S_b	13.9	28.4	+ 6	− 13	330	660	
1917	6946	S_c	11.1	25.3	− 30	−105	610	610	16

Limits in Distance (parsecs)	0–300	301–600	601–900	901–1200
Numbers of Supernovae	3	6	6	5

The angular distances (nova–galaxy nucleus), given in columns six and seven, were obtained from the sources indicated by the references in column ten or were measured by the author from Harvard plates. The distance moduli ($m-M$), apparent magnitudes (m_{pg}) and types for the galaxies were, when available, adopted from Baade's[17] tabulation. In other cases an absolute magnitude of -14.5 was assumed[18] and Miss Sawyer's[19] values of apparent magnitude were used. The year in which the supernova attained maximum light is given in the first column. A questionable supernova which has been omitted from table 1 is the one discovered by Wolf[20]

near Messier 101. Its absolute magnitude did not certainly exceed -10 and the light curve was peculiarly flat. Miss Sawyer omitted it from her list. The computed value of r_c is 2060.

The numbers of supernovae for equal steps in nuclear distance are shown at the bottom of table 1. It can be seen that the frequency is fairly uniform with distance, indicating approximately a $1/r$ law of frequency against area in the principal plane of the average galaxy. The surface luminosity is distributed in a similar fashion. It is of importance to notice that few supernovae occur outside the "main bodies" of the galaxies as measured by Hubble.[21] The Crab Nebula appears to have been produced by a supernova[22] and is an exception in regard to position, as would be Wolf's supernova in Messier 101, if real.

A study of the literature and photographs shows that ten of the twenty supernovae listed above were apparently located on the main axis, a conspicuous arm or a condensation of the galaxy. Eight were in areas of intermediate brightness or could not be well studied, and only two were in apparently dark areas. Zwicky[8] states that for N. G. C. 4157 the supernova occurred in an obscured area. Supernova 4321 (1914) occurred outside the brighter regions but was observed on only one plate. As a working hypothesis it seems likely that the supernovae are distributed in an external galaxy as is the luminosity, with perhaps some tendency to avoid the nucleus. Hubble[23] came to this conclusion for the ordinary novae in Messier 31. The quantitative distribution of supernovae in linear distance from the nuclei is remarkably similar to that for the ordinary novae as observed by Hubble.

2. *Theory of Collisions.*—The frequency of stellar collisions may be calculated in the same manner as the frequency of molecular collisions in a gas, except that the effective target area for stellar collisions depends upon the relative velocity of the stars at infinity, as Schwarzschild,[24] Jeans[2] and others have mentioned. For two stars of equal mass, m_0, with relative velocity, v_∞, the radius, σ, of the effective target area (corresponding to the diameter of a molecule in the gas theory) to allow a periastron distance of q_0 between centers is given by

$$\sigma^2 = \frac{4Gm_0q_0}{v_\infty^2} + q_0^2 \qquad (1)$$

where G is the constant of gravitation and c. g. s. units are employed. For dwarf stars the q_0^2 term is negligible for actual collisions and will be omitted hereafter.

If we assume a Maxwellian distribution of stellar velocities and integrate over all relative velocities, the classical expression for the number of collisions is changed to

$$\frac{\text{Number of Collisions}}{\text{cm.}^3 \text{ sec.}} = 4\sqrt{2}\, Gm_0q_0\, N_0^2/v_0 \qquad (2)$$

where N_0 is the number of stars per cm.^{3} and v_0 is the arithmetic mean velocity.

If the mass of the sun is adopted as unity, N = number of stars per cubic parsec, v = arithmetic mean velocity in km./sec. and q is taken in units of the *radius* of the sun, the expression becomes

$$\frac{\text{Number Collisions}}{(\text{parsec})^3 \text{ year}} = \frac{N^2mq}{v}\,[-15.251], \qquad (3)$$

where brackets indicate a logarithm to the base 10. If a galaxy contains M solar masses uniformly distributed over a volume of V cps., $N = M/mV$, and we find

$$\frac{\text{Number Collisions}}{\text{Galaxy year}} = \frac{M^2q}{mvV}\,[-15.251]. \qquad (4)$$

3. *Collisional Frequency.*—In the application of equation (4) the most uncertain quantity is probably the mass of the main body of a galaxy. We shall adopt values for the other quantities from observation, and calculate collisional frequencies for various estimates of the mass. The average volume may be taken as that of a disc with radius 1000 parsecs and thickness one-third the radius. $V = [+9.02]$ (parsecs).^{3}

The average star is probably of low luminosity and small mass, in accordance with the low mass-luminosity relation for external galaxies and particularly in view of the prevalence of faint stars in our local system. We adopt an absolute bolometric magnitude for the average star of $+10$ to $+11$, in rough agreement with Luyten's unpublished results. According to Kuiper[25] the corresponding mass may be taken as $^1/_{10}$ the solar mass and the radius $^1/_3$ the solar radius.

The average stellar velocity should refer to the center of gravity of each local region of the galaxy. According to the *virial* theorem, the velocity should increase with the space density and with the volume assumed to represent local conditions. We adopt $v = 100$ km./sec. as a representative value.

With these adopted values of V, m, q and v, the number of stellar collisions (limb to center) per galaxy per year is calculated in table 2 for three assumed masses of the main body of an average galaxy. A mass of 5×10^9 suns is considered a minimum value, while 5×10^{10} suns represents perhaps the best present-day estimate.[21,26] A mass of 2×10^{11} suns appears to be an overestimate for the main body of a galaxy, though possibly a fair approximation for the entire system.[27]

For purposes of comparison, the relative frequencies of calculated colli-

sions to observed supernovae are given in the last column of table 2. Zwicky's[28] determination that, on the average, one supernova occurs in a galaxy in six hundred years is adopted.

TABLE 2

COLLISIONAL FREQUENCIES*

TOTAL MASS (SUNS/GALAXY)	DENSITY (SUNS/CU. PARSEC)	COLLISIONS (GALAXY/YEAR)	$\frac{\text{COLLISIONS}}{\text{SUPERNOVAE}}$
5×10^9	5	0.0000005	0.0003
5×10^{10}	50	0.00005	0.03
2×10^{11}	190	0.0007	0.4

* For stars like the sun ($m = q = 1$), multiply frequencies by 0.3.

It can be seen from table 2 that under the most extreme conditions the frequency of collisions is comparable with the number of supernovae. The agreement is reduced by a factor of three if the average star is taken to be of the same mass and radius as the sun.

The slight tendency for the supernovae to avoid the nuclei of galaxies, where one might expect the greatest number of collisions, may be explained on the basis of absorbing matter and observational selection. One must note too that the collisional frequency varies inversely as the mean velocity and that the velocity will certainly increase toward the nuclei, thus tending to counteract partially the effect of high central densities.

4. *Energy Considerations.*—The total energy lost by a supernova is somewhat uncertain. If the energy distribution approximates that of the sun, the loss by radiation varies from about 1×10^{47} to 4×10^{48} ergs for the various supernovae.[29] A correction of two to allow for an A-type distribution would probably be sufficient, if the supernovae spectra are similar to novae spectra except for the line widths; otherwise, the factor would probably be larger.[30] In addition, there is an unknown amount of energy lost by the ejection of material at high velocities. It seems probable, at present, that a correction factor of 5 to 10 should allow for both ejection and radiation. In a nova the energy of ejection is smaller than the energy of radiation. We shall assume that the total energy to be accounted for may lie between 10^{48} and 4×10^{49} ergs for the various supernovae.

Two stars similar to the sun in mass and radius would possess 4×10^{48} ergs of kinetic energy at the instant of an edge-to-center encounter. Although this amount of energy is in excellent agreement with that probably released by a supernova, it seems unlikely that a large fraction of the kinetic energy can be converted into radiation. A discussion similar to that by Jeffreys[31] shows that the conservation of angular momentum will force the stars to pass by each other with a small energy loss. It is possible, however, that under the conditions assumed above, five per cent of the

kinetic energy may be converted. For stars of $^1/_{10}$ the sun's mass and $^1/_3$ its radius, the energies calculated above are reduced by a factor of 30.

It should be noted that the kinetic energy at the time of collision for stars of equal mass varies *inversely* as the distance of their centers, and that the probability of collision varies *directly* as the distance. In addition, the percentage of the kinetic energy that may be converted into radiation should *increase* with decreasing periastron distance both because of the decreasing amount of angular momentum and because the stars are highly concentrated toward their centers. Thus there will be frequent cases in which the energy release exceeds by a considerable amount the values calculated above.

The effect of the high central concentration of stars is very important. For values of periastron distance above a certain critical value (perhaps 0.6 to 1.0 radius for the Emden polytrope of index 3), the quantity of matter involved in the collision decreases rapidly, as does the amount with negative angular momentum about the center of gravity. Thus the total energy that can be converted into radiation and therefore the total visible radiation of the outburst quickly become negligible with increasing periastron distance. For example, suppose the critical distance were $q = 1.0$ for solar masses and the corresponding radiant energy, 2×10^{47} ergs. Then one should expect an equal number of supernovae with energies above and below this value. The maximum possible energy for solar masses is possibly of the order of 2×10^{49} ergs. Under these assumptions one-half of the supernovae would occur within a range of five magnitudes, the other one-half being fainter over an infinite range. With a mixture of stars of various masses the critical range would be increased but the general effect would be similar to that observed, a concentration near the magnitude of maximum frequency.[17,19]

Discussion.—It is not the purpose of the present paper to compare the various possible hypotheses for the origin of supernovae but to show that the collision hypothesis deserves consideration. There are several phases of the hypothesis that must be critically analyzed before a judgment of its merits can be made. In particular, there is the problem of whether an actual collision would produce the observed phenomenon. In the last section it was assumed that much of the energy from an inelastic collision would be released in the form of radiation. This assumption is open to question until the detailed processes involved are carefully studied. In any case, the equilibrium of the stars must be seriously disturbed by a collision, and it is possible that considerable energy may be released as new states of equilibrium are attained. Eddington,[32] for example, has shown that a star has a store of energy comparable to the amounts assumed for supernovae, while Milne[33] has suggested that novae result from the collapse of stellar cores and Baade and Zwicky[34] that supernovae result in

neutron cores. Regardless of the exact process by which the energy is released it is important to note that *the energy will always be associated with matter*, and that the matter will act, by its opacity and expansion, as a blanket to excessively high temperature radiation until some sort of equilibrium is reëstablished.

Another critical point concerns the total energy actually released by a supernova. This quantity is chiefly a matter of opinion until the fundamental characteristics of the spectra are ascertained. The problem is complicated by the ejection of gases at high velocities in the supernova phenomenon.

The problem of the frequency of collisions to be expected in an external galaxy is far from an exact solution as yet. Not only is the mean density of matter uncertain, but the distribution function of stellar masses is practically unknown. In addition, a very careful study of the velocity distribution must be made when the mass functions and densities are better determined.

In spite of these uncertainties, the collision-hypothesis shows promise of providing a logical explanation for the very remarkable phenomenon of the supernovae.

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OBSERVATIONS ON CHROMOSOME ELIMINATION IN THE GERM CELLS OF SCIARA OCELLARIS[1]

BY R. O. BERRY

DEPARTMENT OF ZOÖLOGY, JOHNS HOPKINS UNIVERSITY

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One of the interesting problems that have arisen in the course of the work with Sciara concerns the time and manner of chromosome elimination from the germ line of this organism. Since the sperm regularly transmits an extra sex chromosome to the fertilized egg (Metz, Moses and Hoppe, 1926),[2] it necessarily follows that a subsequent elimination process must occur in order to maintain a constant chromosome number in the germ line (for general review see Metz, 1938).[3] The sperm contributes five chromosomes, one more than the haploid number, while the egg has the normal haploid number of four. Thus in the fertilized egg there are nine chromosomes. However, observations on the gonads of the early larval stage show that there are only eight chromosomes present. Therefore, one chromosome must have been eliminated at some early stage of development. It is the purpose of this paper to give some of the details of the process of elimination as it occurs in the germ cells.

DuBois (1932, 1933)[4] observed chromosome elimination from the somatic cells of *Sciara coprophila* during the early cleavage stages. In this species the "limited" chromosomes were eliminated at the fifth cleavage, and one ordinary chromosome in the female and two ordinary chromosomes in the male were eliminated at the seventh or eighth cleavage of the cells of the somatic line. In each case the eliminated chromosomes failed to complete the mitotic process and remained at the equatorial plate, thereby being excluded from the daughter nuclei. DuBois also found that the germ cells had migrated into the poleplasm previous to the time of chromosome elimination in the somatic cells, but her observations did not reveal any elimination from the cells of the germ line at that time.

In the present study observations on the chromosomes of the germ cells of *Sciara ocellaris* Comst., have been made from the time of their differentia-

tion to the formation of the gonads. These observations indicate that there is no chromosome elimination during the cleavage stages of the germ cells that is comparable to the process which occurs in the somatic cells, but that there is an elimination of one chromosome from the germ cells of both the male and female embryos after these cells have migrated to their future gonadal site. This elimination occurs from a resting cell and is apparently effected by the moving of the chromosome from its position within the nucleus directly through the nuclear membrane into the cytoplasm. The elimination occurs at a definite stage in embryonic development and approximately at the same time in all the germ cells of a single individual.

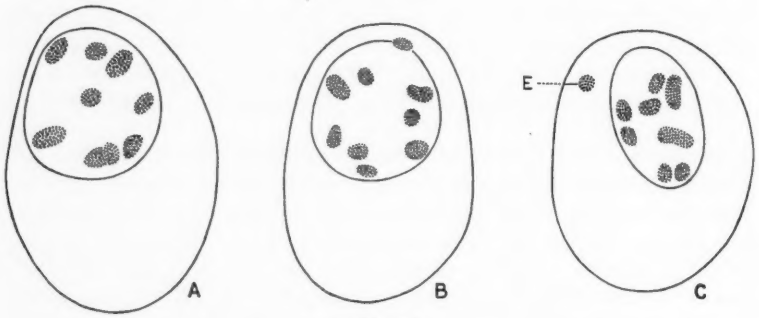

FIGURE 1

Camera lucida drawings from aceto-carmine preparations of cells in the germ line. × 1800.

A. A germ cell before migration, showing nine chromosomes.

B. A germ cell in which the one chromosome is being eliminated from the nucleus.

C. A germ cell after elimination has occurred. There are eight chromosomes in the nucleus and one in the cytoplasm (E).

The eliminated chromosome remains in the cytoplasm for several days and then gradually degenerates.

Within about seven hours after the eggs are laid, the germ cells complete their division stages at the pole and subsequently go into a resting stage (Fig. 1,A). This resting stage is maintained throughout the period of germ cell migration and for some time thereafter. The absence of mitotic activity in this period is shown not only by the observed absence of mitotic figures but also by the fact that the number of germ cells formed at the pole before migration occurs is equal on the average to the number found in the very young gonads. The chromosomes in the resting cells are in the form of diffuse but definite bodies ("prochromosomes"). There are nine of these in the nucleus of the early germ cells, but at a period in development when the caudal end of the embryo has completely surrounded the germ cell, one chromosome passes out into the cytoplasm of each germ cell. There-

after one chromosome is found in the cytoplasm and eight in the nucleus (Fig. 1,C). The body in the cytoplasm can be identified as a chromosome not only by its morphological features but also by the fact that it stains with the usual nuclear stains and gives a positive Feulgen reaction. This elimination is a regular process that occurs in the germ cells of all the developing embryos.

The exact mechanism involved in the process of elimination is not known. Observations just previous to the time of elimination do not reveal any characteristics to distinguish the chromosome which is to be eliminated from any of the others. This chromosome apparently does not undergo a change, nor does the nucleus as a whole appear changed in any way to facilitate the exit of the chromosome. There is no budding and the nuclear membrane appears to be present at all times. Numerous observations on stages just previous to and during elimination indicate that the chromosome that is to be eliminated at first comes into close contact with the nuclear membrane and then migrates through it into the cytoplasm (Fig. 1,B). Its movement through the nuclear membrane seems to be autonomous, but the activity is probably affected by some coördinating influence, since the process of chromosome elimination occurs in all the germ cells of a single individual at approximately the same time and in all individuals at a particular period in their development.

The eliminated chromosome cannot be distinguished morphologically from the other six rod-like chromosomes of the group, but genetic evidence shows that it is one of the two sister sex chromosomes contributed by the sperm (Metz, 1938).[3]

So far as the writer is aware this is the only instance known in which a chromosome is eliminated from the germ line, except during maturation, and is the only instance of the occurrence of a chromosome elimination from a resting cell. The bearing of the phenomena on the problem of chromosome movement and on sex determination will receive consideration elsewhere.

[1] The author is indebted to Dr. C. W. Metz, Department of Embryology, Carnegie Institution of Washington, for helpful criticism during the course of this work. The author is now at the Agricultural Experiment Station, A. and M. College of Texas.

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RELIABILITY OF MEASUREMENTS OF THE STEADY POLARITY POTENTIAL OF THE EYE*

BY WALTER R. MILES

LABORATORY OF PHYSIOLOGICAL PSYCHOLOGY, YALE UNIVERSITY

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Currents of rest and action currents are both demonstrable and measurable from the human eye. The action potential follows on stimulation of the eye with light. The resting potential appears as a steady polarization phenomenon characteristic of the eye as a living system of tissues. We know how to go about producing an action potential, but at present we have no clearly outlined technique for varying the much larger steady potential. Both phenomena are readily measurable in millivolts by the technique of applying metal foil electrodes to the skin next the eyes for connecting the subject, through a direct current vacuum tube microvoltmeter, to a sensitive galvanometer such as the Einthoven.[1] After having compensated other body potentials the ocular polarity potential is found when the eye has moved laterally or vertically from a central fixation point. Normally the potential shown on the galvanometer remains steady as long as the fixation of the eye holds on any peripheral position.

In a former paper[2] the steady potential of the human eye was examined by the method just indicated and results were presented on a group of 15 normal subjects measured for lateral eye rotations of 30°. In the present paper data will be presented bearing on five phases of the ocular polarity problem: (1) the reliability of measurements of the steady potential of the human eye; (2) individual differences and variabilities in normal subjects; (3) the question of an age gradient; (4) relationship of the eye's steady potential to the potentials of nearby tissues; and (5) electrode lead relationships and their implications.

The data discussed here were secured from individual studies on 56 normal females: girls, young women and mature women, all of whom were paid for their services as subjects. There were 20 Girl Scouts between the ages of 10 and 12, 20 normal school girls, 17 to 19, and 16 older women between the ages of 41 and 65. An additional group of 14 normals, 3 women and 11 men, representing the age range from 21 to 37 years, also served as volunteer subjects. The first examination required about one hour. In addition to eye potentials registered from 10 different electrode lead combinations, the observations included pulse rate, blood pressure, eye dominance, several visual tests, some physical measurements and queries about health and physiological state. Aside from the registration of eye potentials the entire examination was conducted by a young woman laboratory assistant who is a trained nurse. There was nothing painful or frightening about the ex-

amination and every effort was made to dismiss any feelings of apprehension the subjects might have. The second examination, which came some days later, was briefer and could be accomplished in a half hour session.

The reliability of ocular polarity potential measurements may be judged from comparisons of the first and second series of values found for the total group of 56 females and for the three groups considered separately. The statistical values for 5 of the lateral leads on the 56 cases are shown in table 1. The placement of the electrodes will be clear from figure 1. Lead 1 was across the left eye; Lead 2 was across the right eye; Lead 3 was on the

TABLE 1

MEAN VALUES AND CORRELATIONS OF OCULAR POLARITY POTENTIALS MEASURED FROM LATERAL EYE ROTATIONS OF 30 DEGREES ON 56 NORMAL FEMALE SUBJECTS*

LATERAL LEADS	FIRST SERIES		SECOND SERIES		CORRELATIONS†	
Ld. 1	M_1	0.81	M_2	0.78	N	55
Left Eye	S. D.	0.19	S. D.	0.17	r	+0.60
	P. $E_{.M}$	0.017	P. $E_{.M}$	0.015	P. $E_{.r}$	0.059
Ld. 2	M_1	0.87	M_2	0.81	N	56
Right Eye	S. D.	0.24	S. D.	0.19	r	+0.56
	P. $E_{.M}$	0.022	P. $E_{.M}$	0.017	P. $E_{.r}$	0.062
Ld. 3	M_1	1.22	M_2	1.14	N	55
Temples	S. D.	0.30	S. D.	0.25	r	+0.75
	P. $E_{.M}$	0.027	P. $E_{.M}$	0.023	P. $E_{.r}$	0.040
Ld. 4	M_1	0.55	M_2	0.49	N	54
Nasals	S. D.	0.15	S. D.	0.13	r	+0.39
	P. $E_{.M}$	0.014	P. $E_{.M}$	0.012	P. $E_{.r}$	0.079
Ld. 5	M_1	0.84	M_2	0.77	N	56
Parallel	S. D.	0.19	S. D.	0.17	r	+0.46
	P. $E_{.M}$	0.017	P. $E_{.M}$	0.015	P. $E_{.r}$	0.072

* The values in the columns headed "First Series" and "Second Series" are in millivolts.

† Pearson product-moment coefficients.

temples; Lead 4 involved the use of the two nasal electrodes; and Lead 5 was an arrangement which placed the two eyes in parallel electrical connection for lateral recording. The results obtained from use of the electrodes above and below each eye will not be considered in the present brief report.

The mean values for polarity potentials as recorded on the first day are shown at the left in table 1, those for the second day in the center and the correlation coefficients with their probable errors are exhibited at the right of the table. The different leads give different potentials and therefore must be treated separately. Lead 3 on the temples gives the largest potential, a value which is about $^1/_3$ higher than the average of Leads 1 and 2.†

Lead 4, the nasal lead, shows an average potential about $^1/_2$ that of Lead 3, and $^2/_3$ the average of Leads 1 and 2. Lead 5, the parallel connection, gives an average potential closely equal to the mean of Leads 1 and 2. The correlation coefficients for Leads 1 to 5, respectively, are: +0.60, +0.56, +0.75, +0.39, +0.46 and all the P. E.'s are small. Lead 3, with +0.75 ±0.04, demonstrates a good degree of reliability. On Lead 3 there were 35 per cent of the subjects who gave pairs of values in which the second measurement deviated from the first by not more than 10 per cent. An additional 43 per cent of the group showed individual deviations for first and second measurements on Lead 3 of from 12 to 20 per cent; and $^1/_5$ of the subjects had higher deviations with one extreme case deviating 40 per cent. A comparison of results for Lead 3 for each of the sub-groups of females, and for the added group of 14 mixed normals, males and females, is

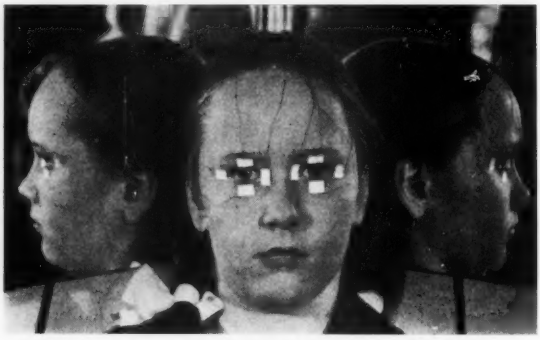

FIGURE 1

given in table 2. Even in these small groups of 20 or less subjects the coefficients were all plus, and in 3 instances show reliability values of about +0.80 ±0.07. The general reliability of the eye potential measurements is further indicated by the positive and moderately high correlation coefficients found between the values obtained on different leads on the first day. Leads 1 and 2 for the 56 females measured in their first examination gave a coefficient of +0.68 ±0.049. The others were as follows: Leads 1 and 3, +0.83 ±0.028; Leads 2 and 3, +0.80 ±0.032; Leads 3 and 4, +0.42 ±0.076; Leads 1 and 4, +0.59, ±0.06; Leads 2 and 4, +0.64 ±0.05; Leads 1 and 5, +0.66, ±0.05; Leads 2 and 5, +0.76 ±0.04; and Leads 3 and 5 +0.70 ±0.05. Some of the coefficients are spuriously high because they involve one electrode in common. It is noteworthy that coefficients for Lead 4, the nasal electrodes, tend to be low.

The correlations between series of measurements and between different leads point to the conclusion that the steady eye potential may be con-

sistently high or low in different supposedly normal subjects, and thus lend justification to the effort to discover variables accompanying these wide differences. It seems probable that there are several factors which may influence the eye's steady potential. At present it is not clear what should be controlled in seeking to duplicate potential measurements on different days. The reliability coefficients that have been found must therefore be regarded as first approximations.

There appears to be a psychological factor relating to familiarity with the measurement routine that has a small but regular influence on the re-

TABLE 2

COMPARATIVE MEANS AND CORRELATIONS OF OCULAR POLARITY POTENTIALS MEASURED ON FOUR GROUPS OF NORMAL SUBJECTS BY LEAD 3 FROM LATERAL EYE ROTATIONS OF 30 DEGREES*

SUBJECTS	FIRST SERIES		SECOND SERIES		CORRELATIONS†	
	M_1	1.17	M_2	1.11	N	20
Girl Scouts	S. D.	0.32	S. D.	0.32	r	+0.81
	P. E.$_M$	0.048	P. E.$_M$	0.048	P. E.$_r$	0.052
	M_1	1.37	M_2	1.20	N	20
Young Women	S. D.	0.29	S. D.	0.21	r	+0.77
	P. E.$_M$	0.044	P. E.$_M$	0.032	P. E.$_r$	0.062
	M_1	1.10	M_2	1.09	N	15
Mature Women	S. D.	0.21	S. D.	0.17	r	+0.59
	P. E.$_M$	0.036	P. E.$_M$	0.030	P. E.$_r$	0.11
	M_1	1.07	M_2	0.99	N	14
Mixed F & M	S. D.	0.29	S. D.	0.22	r	+0.78
	P. E.$_M$	0.052	P. E.$_M$	0.039	P. E.$_r$	0.072

* The values in the columns headed "First Series" and "Second Series" are in millivolts.

† Pearson product-moment coefficients.

sults. In each sub-group, for Lead 3 as shown in table 2, the mean for the second day is lower than for the first. The difference for the normal school girls is M_1–M_2 = 0.17 mv., P. E. diff. = 0.054 and C. R. = 3.1. But the other groups do not show as large differences. The same tendency for a decrease in the means (and also in the standard deviations and probable errors of the means) shows in the other leads as is to be seen in table 1. The obvious conclusion from such a general trend is that the subjects were slightly apprehensive at the time of the first examination. A second time subjects came there was less tension and the experience was more a mere matter of routine. The psychological formulation suggested is in line with what has been found for basal metabolism determinations and other

measurements under laboratory environments strange to the subjects examined.

The individual differences in the ocular polarity potential, as measured by Lead 3, within the different sub-groups of subjects are shown in terms of total range in table 3. The lowest value was 0.45 and the highest 2.20 mv., giving a ratio of 4.9. Within the subject groups the ratios are usually just above 2. The Girl Scouts reveal the lowest values and the Scouts and young women give top values which tend to exceed those of the mature women and also of the mixed group. Adequate data are not available for comparison between males and females, and for the present the individual differences which have been found must remain uncorrelated with other physiological or psychological traits. Elucidation of the cause of these individual differences, and indeed of the steady potential itself, will no doubt necessitate much further research.

TABLE 3

RANGE OF INDIVIDUAL DIFFERENCES IN LEAD 3 POTENTIALS*

SUBJECTS	SERIES	POTENTIALS LOWEST	POTENTIALS HIGHEST	RATIO
Girl Scouts	1	0.45	1.90	4.2
	2	0.56	1.63	2.9
Young Women	1	0.98	2.20	2.2
	2	0.74	1.61	2.2
Mature Women	1	0.73	1.49	2.0
	2	0.83	1.38	1.7
Mixed F & M	1	0.70	1.55	2.2
	2	0.57	1.28	2.2

* Values in millivolts.

The selection of the three sub-groups of females was of course a matter of experimental design to aid in examining for the presence of an age gradient. The averages for Lead 3 as exhibited in table 2 show comparative results typical of the differences found for the three groups in all their leads. The evidence may be interpreted as favoring the hypothesis of a curvilinear relationship between the steady potential of the eye and chronological age with the high point coinciding with late adolescence. The young women gave the highest mean values, the Girl Scouts were second in rank and the mature women averaged lowest for the three groups. In the first series the difference between the means for the Scouts and the young women amounted to 0.20 mv., P. E. diff. 0.065, C. R. 3.1; and for the young women and the mature women the difference was 0.27 mv., with P. E. diff. 0.057 and C. R. 4.7. However, in the second series the differences were

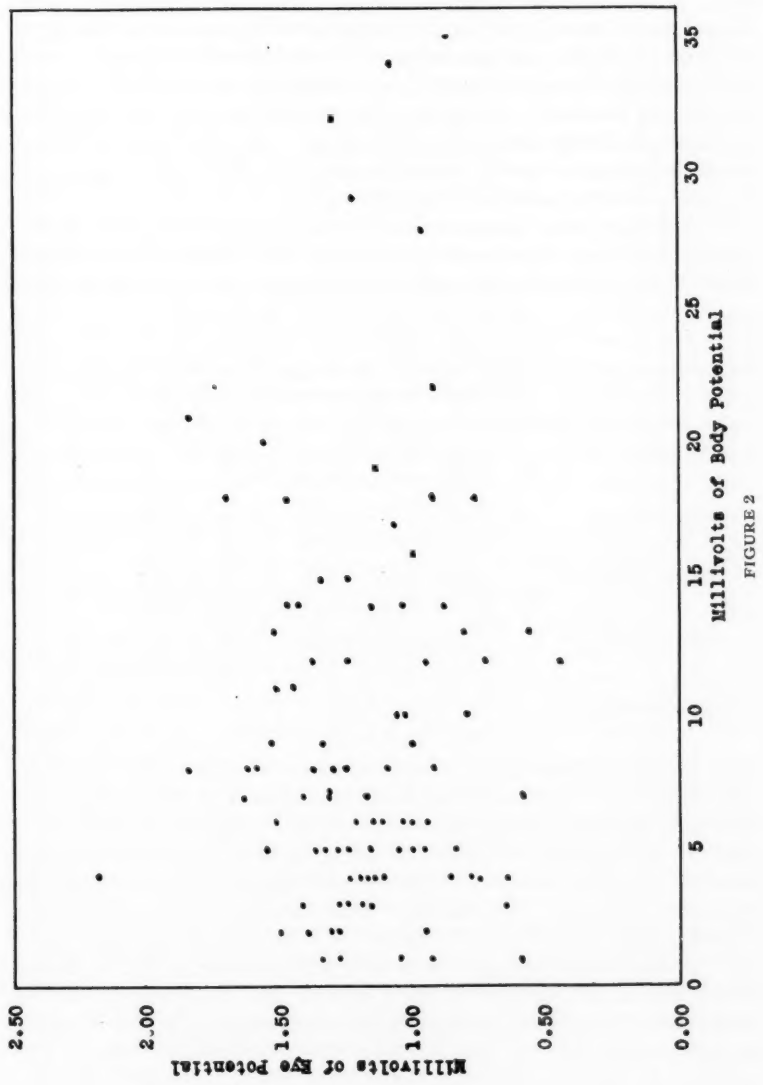
Millivolts of Body Potential
0
5
10
15
20
25
30
35
Millivolts of Eye Potential
2.50
2.00
1.50
1.00
0.50
0.00

FIGURE 2

smaller and the critical ratios turned out to be only 1.6 and 2.5, respectively. As it seems very likely that the second series was the more dependable, in that it was less subject to extraneous influences of a psychological character, it may be assumed at present that chronological age of itself, from early adolescence to middle life, exercises no great influence on the eye's steady potential. There may, however, be an early small positive and a late small negative correlation with age. It is noteworthy that the standard deviations show a consistent age trend; they were largest with the youngest and smallest with the oldest group.

The skin and other tissues surrounding the eye ordinarily show potentials that have to be compensated by a counter e. m. f. before the steady potential of the eye can be isolated for measurement. Subjects show much

TABLE 4

RELATIONSHIP BETWEEN MEAN VALUES FOR STEADY POTENTIALS MEASURED BY DIFFERENT ELECTRODE LEADS*

SUBJECTS	SERIES	LD. 1	LD. 2		LD. 3	LD. 4
Girl Scouts	1	0.78	+ 0.83	= 1.61	1.65 =	1.17 + 0.48
	2	0.78	+ 0.75	= 1.53	1.57 =	1.11 + 0.46
Young Women	1	0.90	+ 0.96	= 1.86	1.97 =	1.37 + 0.60
	2	0.81	+ 0.88	= 1.69	1.72 =	1.20 + 0.52
Mature Women	1	0.75	+ 0.82	= 1.57	1.67 =	1.10 + 0.57
	2	0.76	+ 0.78	= 1.54	1.60 =	1.09 + 0.51
Mixed F & M	1	0.72	+ 0.79	= 1.51	1.53 =	1.07 + 0.46
	2	0.74	+ 0.67	= 1.41	1.42 =	0.99 + 0.43

* Values in millivolts.

larger individual differences in their body potentials than in the eye potentials themselves. Sometimes the body potentials have one polarity and sometimes the opposite, but the polarity of the eye appears to remain unchanged in direction. Occasionally, on some one lead, body potential seems to be absent and not a single millivolt need be applied to bring the galvanometer to zero. On the other hand there have been patients who required as much as 100 mv. of counter e. m. f. to balance their body potential. Ordinarily the body potentials hold fairly constant, or if not, change slowly, and do not seriously interfere with the eye measurement. If the subject should be startled, body potentials change rapidly, as is well known to psychologists through their work on the psychogalvanic reflex. The question of a possible interaction between the steady potential of the eyeball and the relatively steady potentials of tissues near the eye naturally suggests itself. The charted data shown in figure 2 provide a tentative answer to this question. Eye potential has been used as ordinate and body

potential as abscissa. The plotted observations indicate no clear tendency for either positive or negative correlation between the two potential measurements. But this conclusion must be qualified. Balancing by counter e. m. f. should produce the equivalent of zero body potential at the electrodes, and the eye potential is measured under this condition. Just before balancing, body potential has been present. The results at hand do not show what may have been the relation at the moment before balancing. Perhaps the eye swiftly recovers from any influence that the polarized tissues around it exert; this possibility must be left open at present.

If the eyes were insulated electrically from surrounding tissues their potentials could not be measured by the method described. As it is, placement of a pair of electrodes near the eyes shows the presence of potentials in them if the eyes take up peripheral fixation positions: (1) when one eye is apparently included between the pair of electrodes, and (2) when one electrode is in proximity with one eye and the other electrode is nearer the other eye. The tissues surrounding the eyes are conducting dielectrics and therefore the two eyes cannot together pile up a potential at the electrodes greater than either eye itself can supply, since there is no way of holding this potential from leaking out. While the two eyes seldom exactly match each other in steady potential, still the pair of eyes creates an electrical field, the potential of which can be tapped at different places, and according to the place where it is tapped a value is obtained which has a certain relationship with the values found from tapping the field at other points. Previously[2] attention was called to the equation: Lead 1 + Lead 2 = Lead 3 + Lead 4. Applying this to the data for lateral leads presented in the present paper gives the results shown in table 4. There are 8 comparisons available. The two center columns in the table give the sums of the respective pairs of leads. In every instance the Ld. 3 + Ld. 4 combination is slightly higher, on the average 3 per cent. The relationships may be put in another way by writing: Lead 1 + Lead 2 − Lead 4 = Lead 3; in this form the results are as shown in table 5. The calculated and actual (as measured) Lead 3 values tend to agree closely, but the measured potential on Lead 3 for each subject and session averages a little higher than the calculated. Possibly this difference is related to the order in which the leads were taken in the experimental session, and is another instance of slight tension at first and greater relaxation later. Lead 3 was in fact always taken first, Lead 2 came second, Lead 1 was third, Lead 4 was fourth and Lead 5 was recorded fifth. Incidentally it may be noted that Lead 2 tends to be larger than Lead 1. A balanced order to rule out such possible influences of experimental routine is obviously desirable in future work.

Conclusions.—1. Ocular polarity potentials, measured without voltage drop due to current drainage or electrode resistance, are reported on three groups of normal females: 20 Girl Scouts age 10–12, 20 normal school

girls age 17–19, and 16 mature women 41–65 years old, and for a group of 14 mixed normals ranging in age from 21 to 37. Ten different eye potential measurements were made, and each subject served on two different days.

2. The correlation coefficients for five lateral leads are reported for the entire group of 56 cases. All the coefficients are positive and range between $+0.39 \pm 0.08$ and $+0.75 \pm 0.04$. The latter coefficient is from Lead 3 across the temples, which gives the highest potential. On Lead 3 the Girl Scouts gave a coefficient of $+0.81 \pm 0.05$, the normal school girls $+0.77 \pm 0.06$ and the 16 mature women showed $+0.59 \pm 0.11$. The extra group of 14 (males and females) on Lead 3 gave $+0.78 \pm 0.07$. The coefficients indicate that the ocular polarity potential measurement is a fairly reliable one. Half of the subjects gave values for Lead 3 on the two different days that were within 15 per cent or less of each other.

TABLE 5

PREDICTION OF THE POTENTIAL FOR LEAD 3 FROM MEASUREMENTS MADE ON OTHER LEADS*

SUBJECTS	SERIES	LD. 1	LD. 2	LD. 4	LD. 3 CALCULATED	LD. 3 ACTUAL
Girl Scouts	1	0.78	+ 0.83	− 0.48 =	1.13	1.17
	2	0.78	+ 0.75	− 0.46 =	1.07	1.11
Young Women	1	0.90	+ 0.96	− 0.60 =	1.26	1.37
	2	0.81	+ 0.88	− 0.52 =	1.17	1.20
Mature Women	1	0.75	+ 0.82	− 0.57 =	1.00	1.10
	2	0.76	+ 0.78	− 0.51 =	1.03	1.09
Mixed F & M	1	0.72	+ 0.79	− 0.46 =	1.05	1.07
	2	0.74	+ 0.67	− 0.43 =	0.98	0.99

* Values in millivolts.

3. Rather marked individual differences were found in the steady potential of the eye. Some subjects show potentials three or four times as high as other subjects, and some are much more variable than others from day to day. The causes for these individual differences are still unrevealed. Subjects were found to vary more markedly from each other in body potentials than in eye potentials.

4. The data secured on the three age groups of females may be interpreted as favoring the hypothesis of a curvilinear relationship between chronological age and eye potential, but are not conclusive.

5. The steady potential of the eye gives no indication of correlating positively or negatively with the potentials of the tissues surrounding the eye.

6. The relationships between the different electrode leads, by which steady potentials can be measured in association with 30° lateral fixations, appear to be quite constant in normal subjects. The equation: Ld. 1 + Ld. 2 = Ld. 3 + Ld. 4 was found to hold closely in the averages for each sub-group on each day. There is, however, a consistent tendency for Ld. 3 + Ld. 4 to be a little higher than expected. All comparisons thus far worked out show this tendency which averages +3 per cent. The fact that Lead 3 was the first measurement in the series for each patient on each day may have a bearing on the results.

7. A consistent tendency was found for the steady potential measurements to show a slightly smaller value on the second than for the first day. It is assumed that this change rests on psychological factors of mental attitude and adjustment to the laboratory procedure. This is a significant finding, pointing as it does to a relationship between the strength of the eye's steady potential and the psychological state of subjects coöperating in the experimental routine.

* The expenses of this investigation were met in part by a grant from the Knight Fund of Yale University School of Medicine.

† In the previous paper it was suggested that Lead 3 represents the optimal steady potential average for the two eyes. It now appears also to be the most dependable of the several measures of potential used in these studies.

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ON THE SUBDIVISION OF THE GENUS DROSOPHILA

BY A. H. STURTEVANT

W. G. KERCKHOFF LABORATORIES OF THE BIOLOGICAL SCIENCES, CALIFORNIA INSTITUTE OF TECHNOLOGY

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There is now in progress in several laboratories a comparative study of the genetics of the species of Drosophila. Several authors have pointed out possible deductions concerning the history of the group that may be derived from such studies (e.g., Donald 1936, Sturtevant and Tan 1936, Serebrovsky 1938). If such deductions are to be of value, it is desirable that there be available a satisfactory arrangement of the species into some scheme of classification that can be taken as indicating their degree of genetic relationship. It is the purpose of this note to present the results

of an attempt to derive a classification by a method as free of personal equation as it can be made.

There are forty-two species of Drosophila in culture in this laboratory, and these have all been examined for a series of twenty-seven different characters. These characters, which concern the eggs, larvae, pupae and imagines, and include both structural characters and those having to do with the habits of the animals, are all that have so far been worked out that satisfy the following requirements:

(*a*) Each of the species must be capable of classification for every character. Some characters, such as presence or absence of clouds on the cross-veins, permit of a classification into two groups, one having the character, the other lacking it. Other characters, such as proportions of the sections of wing-veins, or number of branches of anterior pupal spiracles, are numerical. In such cases an arbitrary amount of difference between two species was set as being significant.

(*b*) There must be at least two species in each category concerned—i.e., a character occurring in only one of the available species was not included.

(*c*) Two characters obviously likely to be developmentally closely related, or likely to be affected similarly by natural selection, were not both included. This requirement is difficult to apply, and does admit a large element of personal judgment. The rule here should be to include no doubtful cases.

Having a table showing the condition of all 27 characters in each of the 42 species, two different types of analysis were then carried out. A table was made showing, for each species, the number of characters in which it differed from each of the other 41. This table may be taken as giving a first approximation to the degree of genetic difference between any two species—and therefore to their remoteness of genetic relationship. The accuracy of this index evidently depends on several factors, but it seems legitimate to conclude that most resemblances are due to likeness of genetic constitution, and that separate origins of similar constitutions are not likely to give parallel resemblances in many different characters.

Each of the 27 characters was then compared with each of the others, to determine if the distribution of the two was correlated. Here a striking result was at once evident; there is a single group of correlated characters that splits the available species into two groups, with a few forms having combinations of characters that disagree with the usual correlations. Reference to the species-by-species table shows that all but one of these anomalous types can be connected, through a series showing as few as six differences between successive species, to one or the other of the major groups. *Drosophila duncani* alone fails to show such a series of connections, since it differs from every one of the remaining species by at least nine characters. If the remaining species are separated into two groups,

study of the table shows that the minimum number of differences between two species belonging to different groups is also nine.

The following classification is therefore indicated:

Subgenus *Dasydrosophila* Duda

Ventral receptacle not finely coiled; posterior pair of Malpighian tubes fused to form a ring around the gut; four blunt filaments on the eggs; dark bands on posterior margins of second to fifth abdominal segments not thinner or broken in median dorsal line.

Species: *D. duncani* Sturtevant (Illinois, Ohio).

Subgenus *Drosophila* Fallén

Ventral receptacle finely coiled, resembling a more or less tangled spring; posterior pair of Malpighian tubes fused; three or four tapering filaments on the eggs; dark posterior bands of at least the two basal abdominal segments thinner or broken in median dorsal line.

Species agreeing in all these characters: *D. funebris* Fabricius (type—temperate regions); undescribed species near *funebris* (Texas); *D. guttifera* Walker (southeastern United States); *D. hydei* Sturtevant (American); four undescribed species near *repleta* and *hydei* (Mexico, Guatemala, California); *D. repleta* Wollaston (cosmopolitan); *D. mulleri* Sturtevant (American); *D. immigrans* Sturtevant (cosmopolitan); *D. tripunctata* Loew (southeastern United States) *D. testacea* v. Roser (north temperate zone); *D. busckii* Coquiller (cosmopolitan).

Species diverging in one or more of the characters listed:

Posterior Malpighian tubes apposed, but not with continuous lumen: *D. cardini* Sturtevant (tropical America); *D. robusta* Sturtevant (eastern United States).

Egg filaments blunt: *D. virilis* Sturtevant (Japan, China—rare in the United States).

Egg filaments two, blunt: *D. melanica* Sturtevant and two related species (eastern United States). In one of the undescribed species the ventral receptacle is not finely coiled. Were it not for its close resemblance to *melanica* and to *mulleri* the position of this species would be doubtful.

Subgenus *Sophophora*, subg. nov.

Ventral receptacle not finely coiled; posterior pair of Malpighian tubes free, not united at their apices; eggs with two blunt filaments; dark posterior bands of second to fifth abdominal segments broader in median dorsal line, or of uniform width.

Species agreeing in all these characters: *D. melanogaster* Meigen[1] (type —cosmopolitan); *D. affinis* Sturtevant (eastern United States); *D. algonquin* Sturtevant and Dobzhansky (eastern North America); *D. athabasca* Sturtevant and Dobzhansky (northern North America); *D. azteca* Sturtevant and Dobzhansky (California to Guatemala); *D. narragansett* Sturtevant and Dobzhansky (eastern United States); *D. psuedo-obscura* Frolowa (western North America); *D. subobscura* Collin (Europe); *D. miranda* Dobzhansky (western United States); *D. ananassae* Doleschall (Tropics, old and new worlds); *D. bipectinata* Duda (tropical Asia); *D. auraria* Peng (China, Japan); *D. simulans* Sturtevant (cosmopolitan); *D. takahashii* Sturtevant (Formosa, China); *D. willistoni* Sturtevant (American tropics); six species (probably all undescribed) related to *D. saltans* Sturtevant (Mexico and Central America).

Less complete analysis suggests the following distribution for certain species not now in cultivation in this laboratory:

Subgenus Drosophila: *D. quinaria* Loew, *D. transversa* Fallén, *D. similis* Williston, *D. latevittata* Malloch, *D. californica* Sturtevant, *D. komaii* Kikkawa and Peng, *D. bizonata* Kikkawa and Peng.

Subgenus Sophophora: *D. nebulosa* Sturtevant, *D. obscura* Fallén, *D. seminole* Sturtevant and Dobzhansky, *D. montium* deMeijere, *D. lutea* Kikkawa and Peng.

As indicated, one of the three subgenera has already been recognized by Duda (1925), who has established several subgenera. Two others of these are represented among the species here considered: Acrodrosophila Duda is based on *D. testacea*, and Spinulophila Duda (= Acanthophila Duda) includes *D. immigrans*. Each of these subgenera was based on a single character, and the present analysis does not indicate that the species concerned are particularly out of place in the typical subgenus. Dasydrosophila was based on several characters, the following being a translation of the diagnosis: Third antennal segment very large, at least twice as long as broad and two to three times as long as the second segment, often long haired; facial carina usually low, only exceptionally nose-like; arista with only a single branch below basal to the terminal fork. (In *D. duncani* there are two such branches of the arista.) The type-species is from the Oriental region, and Duda also refers several tropical American forms to this group. I have examined no others for the series of characters here utilized, but it may be supposed that the species associated by the characters given by Duda constitute a natural group.

Unfortunately the distinction between the other two subgenera is not easily made with ordinary museum material. Of the characters given, only the abdominal banding is evident in pinned specimens—and this is often obscured by shrinkage. Other characters may be helpful, as follows: Sex-combs on the male tarsi occur only in Sophophora; species

appreciably larger than melanogaster are found only in Drosophila; the "costal index" (length of second section of costa divided by length of third section) averages lower in Sophophora (range from 1.2 to 3.1) than in Drosophila (2.8 to 4.3, with *D. guttifera* at 2.0); the "sterno-index" (length of anterior sternopleural bristle divided by length of posterior—Kikkawa and Peng 1938) averages lower in Sophophora (0.3 to 0.6) than in Drosophila (0.5 to 0.9, with *D. busckii* at 0.3 and *D. guttifera* at 0.4).

The present account is not to be considered as a final one. Several of the characters used are unsatisfactory, and others are not yet measured accurately. It is hoped that, with improved classification of these and with the utilization of more characters, the method may be extended to indicate still finer subdivisions of the genus. It is also probable that additional species will be studied, and these may require some modifications of the scheme here outlined—though partial analysis of several aberrant types has, so far, suggested that such modifications will consist chiefly in the addition of new subgenera, rather than in the rearrangement of the species here considered.

[1] This is the species known to all geneticists under this name (synonym, *D. ampelophila* Loew). Some authors now use the name *D. fasciata* Meigen. There is some reason for this substitution; but I am not convinced that the argument for it is conclusive. In any case there is no other species to which the name *melanogaster* is properly applicable; and with so widely known and unambiguous a name I am convinced that too close adherence to the strict rules of taxonomy is only pedantic and confusing.

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EMBRYO-SAC DEVELOPMENT IN PLUMBAGELLA

BY J. W. BOYES[1]

DEPARTMENT OF BOTANY, UNIVERSITY OF WISCONSIN

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Dahlgren[2] in 1916 reported on the development of the embryo sac in the Plumbaginaceae. Throughout the family the primary archesporial cell gives rise to a parietal cell and a macrospore mother cell, the latter of which functions as an embryo-sac mother cell. In the sub-family Plumbagineae, of the four nuclei formed by the meiotic divisions two lie at one end of the embryo sac separated from the two at the other end by a large

vacuole (this vacuole appears at the 2-nucleate stage in *Plumbagella*). The nucleus nearest the chalazal end becomes the nucleus of the single antipodal cell; that nearest the micropylar end becomes the nucleus of the egg; and the other two nuclei unite to form a 2 *n* primary endosperm nucleus. In contrast, members of five genera of the sub-family Staticeae examined by Dahlgren form typical 8-nucleate embryo sacs.

Dahlgren's studies included the following members of the Plumbagineae: *Ceratostigma plumbaginoides* Bunge, *Plumbago capensis* Thunb., *P. pulchella* Boiss., *P. zeylanica* L. and *Plumbagella micrantha* (Ledeb.) Spach. *Plumbagella micrantha* was studied more extensively than the other species.

Dahlgren's story has figured prominently in the literature since its publication. Its prominence is due chiefly to the fact that only the meiotic divisions seem to be involved in embryo-sac formation, the gamete (egg) nucleus resulting from the second of these divisions. No similar case of complete reduction of the macrogametophyte is known in any angiosperm.

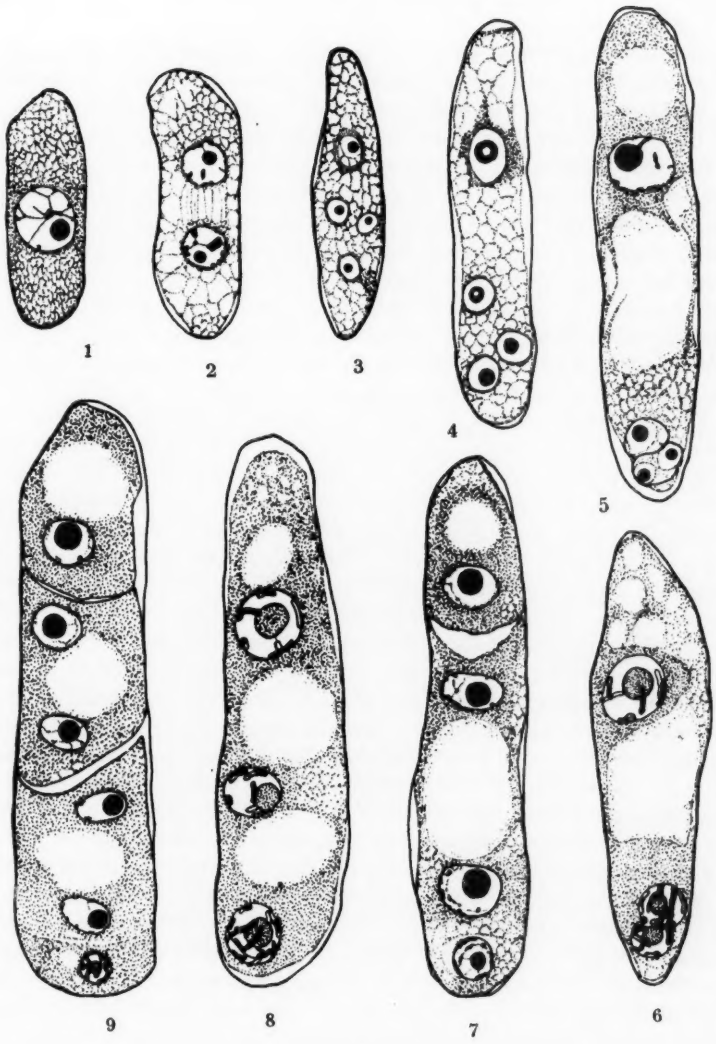
Haupt[3] reinvestigated the development of the embryo sac in *Plumbago capensis*. His story agrees with Dahlgren's up to the 4-nucleate stage. The four nuclei now present divide and the daughter nuclei remain in pairs. Thus an 8-nucleate embryo sac is formed. One micropylar nucleus becomes the egg nucleus. Four nuclei, one from each pair, migrate to the center, where they fuse to give a 4 *n* primary endosperm nucleus.

Dahlgren[4] has more recently concluded that embryo-sac development in *Plumbago zeylanica* and *Ceratostigma plumbaginoides* follows the sequence reported by Haupt for *Plumbago capensis*. Dahlgren considers, however, that his investigation of *Plumbagella micrantha* was so thorough that his report of the history in that species can hardly be in error.

Before the appearance of Dahlgren's paper last cited, the writer had begun studies on embryo-sac development in the Plumbaginaceae. Emphasis has since been shifted to a study of *Plumbagella micrantha*.

According to my observations, the second meiotic division in this species follows immediately after the first in the enlarged macrospore mother cell. Up to this time, no large vacuoles are present. The four nuclei, each with *n* (6) chromosomes, commonly occupy positions corresponding approximately to the four points of a diamond (Fig. 3), although they sometimes have a more nearly linear arrangement. The micropylar nucleus begins to enlarge and very soon is surrounded by a region of dense cytoplasm. The other three nuclei migrate to the chalazal end (Fig. 4). During this period of migration, the embryo sac elongates considerably, a large vacuole appears between the three chalazal nuclei and the micropylar nucleus, and a small vacuole appears between the latter and the micropylar end. The three chalazal nuclei approach each other while the micropylar nucleus continues to grow.

The nuclei in the chalazal region unite in varying degrees of complete-



Figs. 1–7. Development of the usual 4-nucleate embryo sac: 1, macrospore mother cell; 2, 2-nucleate stage; 3, young 4-nucleate stage; 4, migration of 3 nuclei to the chalazal end; 5, two chalazal nuclei uniting, with a third nucleus very close; 6, chalazal nuclei uniting and in prophase; 7, newly formed later 4-nucleate stage.

Figs. 8, 9. Origin of 6-nucleate embryo sacs: 8, 3-nucleate stage with 2 *n* chalazal nucleus; 9, 6-nucleate embryo sac. All figures × ca. 920.

ness. Before the beginning of the prophases, three separate nuclei may be present; two nuclei may have partly united (Fig. 5) or have fused completely, in which case two separate chalazal nuclei enter the prophases; or all three nuclei may have united so that a single 3 *n* nucleus is present.

During the prophases the nuclear fusions already under way continue (Fig. 6). In one case 18 (3 *n*) chromosomes were clearly counted in a single chalazal nucleus. A third series of divisions takes place, with 6 (*n*) chromosomes on the micropylar spindle and 18 (3 *n*) chromosomes on one, or on two closely associated, spindles at the chalazal end. This gives, in either case, a row of only four nuclei; two nuclei at the micropylar end, each having 6 chromosomes; and two at the chalazal end, each with 18. These nuclei are separated from the nucleus of the macrospore mother cell by three divisions, not by two as Dahlgren concluded.

The micropylar nucleus becomes the nucleus of the egg; the nucleus nearest the chalazal end becomes the nucleus of the single antipodal cell (Fig. 7). A period of growth of the embryo sac follows. Then the remaining chalazal nucleus (with 3 *n* chromosomes) migrates toward the micropylar region where it meets and unites with the polar nucleus (having *n* chromosomes) from the micropylar end. At the time of fusion, it is clear that one polar nucleus is larger than the other. Thus a primary endosperm nucleus, having 4 *n* chromosomes, is formed.

About four per cent of the embryo sacs (12 out of 280 observed) become 6-nucleate. From the observation of younger material it would appear that these exceptional cases arise in the following way. After the meiotic divisions, when three nuclei are migrating toward the chalazal end, one nucleus lags behind and is separated from the other two by a large vacuole (Fig. 8). Apparently the nuclei which reach the chalazal end unite, as do the three in the more common case. All nuclei then undergo a division. As a result, the embryo sac has six nuclei, two at the micropylar end, each with *n* chromosomes, two in the central region of the embryo sac each with *n* chromosomes and two at the chalazal end each with 2 *n* chromosomes. In the youngest 6-nucleate embryo sac observed, the nuclei form a longitudinal row, the two nearer the micropyle separated from the four nearer the chalazal end by a large vacuole. The nucleus nearest the micropylar end becomes the nucleus of the egg, the next two nuclei are cut off in a single cell as also are the next two, leaving a single nucleus for the antipodal cell (Fig. 9). Later stages of these 6-nucleate embryo sacs have not been fully studied.

Other exceptional nuclear conditions have been observed, though they are rare. These will be described more fully at a later time; for the present it suffices to say that they are all easily explainable on the basis of the interpretation given above, but seem inexplicable on the basis of the developmental scheme suggested by Dahlgren.

Summary.—1. The egg nucleus in *Plumbagella micrantha* is not, as Dahlgren concluded, one of the four resulting from the meiotic divisions.

2. The egg nucleus is separated from the nucleus of the macrospore mother cell by three divisions.

3. After the second meiotic division three nuclei migrate to the chalazal end of the embryo sac and there commonly unite. One nucleus remains at the micropylar end and divides at the same time that the newly formed chalazal nucleus divides. Of the four resulting nuclei, one (with n chromosomes) becomes the nucleus of the egg, another (with 3 n chromosomes) becomes the nucleus of the single antipodal cell and the other two nuclei (with n and 3 n chromosomes, respectively) unite to form a 4 n primary endosperm nucleus.

4. The present scheme for *Plumbagella* resembles in certain respects that suggested by Haupt and confirmed by Dahlgren for *Plumbago*, especially in the production of a 4 n primary endosperm nucleus.

5. This method of embryo-sac development somewhat resembles that reported for *Lilium* by Bambacioni[5] and Cooper.[6] Three chalazal macrospore nuclei frequently unite in *Plumbagella* before the formation of spindles whereas in *Lilium* the three spindles unite. In *Lilium* there is a fourth division resulting in an 8-nucleate embryo sac, whereas *Plumbagella* remains in the 4-nucleate condition.

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THE DISTRIBUTION OF INTERMEDIN IN THE PARS ANTERIOR OF THE CHICKEN PITUITARY

BY L. H. KLEINHOLZ AND H. RAHN*

THE BIOLOGICAL LABORATORIES, HARVARD UNIVERSITY

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Intermedin regulates melanophore activity in lower vertebrates and recent reports[1] indicate that in mammals this hormone also acts as a specific metabolic principle. Production of intermedin is independent of pars neuralis in amphibians[2] and in pituitaries of those birds[3] and mammals[4] which lack an anatomically distinct pars intermedia, the hormone is found

in the pars anterior. Since the chicken pituitary shows two cytologically distinct lobes (caudal and cephalic) of the pars anterior,[5] a possible correlation was sought between the distribution of intermedin and the localization of specific cell types. Hypophysectomized *Anolis carolinensis*, in which the rate and degree of darkening vary with the concentration of injected intermedin extract, were used in a quantitative study. More extensive reports will be published elsewhere.

Whole pituitaries were rapidly removed from molting hens and immediately immersed in acetone. The dried anterior lobes were divided into 3 approximately equal portions, a cephalic-, middle- and caudal-third. The three lots were extracted by boiling for 10 minutes with N/10 NaOH, neutralized with N/10 HCl and made up with cold-blooded Ringer's to give 3 stock solutions, each containing 0.36 mg. dried powder per 1 0 cc. Quantitative measurements can be made[6] and curves showing the rates of response to various dilutions plotted. The range of melanophore response from complete concentration to maximum dispersion of pigment was divided into 5 stages according to the method devised by Hogben and Slome.[7] A series of progressive dilutions of the stock solutions was made until the injected dose (0.2 cc. per animal, intraperitoneally) evoked an average stage 2 reaction. This concentration was taken as the lower threshold for the integumentary melanophores.

A. Cephalic third-, the maximum degree of melanophore dispersion obtained with the stock solutions, is reached within 15 minutes after injection. Progressive dilutions of the extract result in a marked flattening and displacement of the time curves to the right, i.e., a lower degree of response spread over a longer time period. The concentration which gave an average stage 2 reaction in 10 injected lizards was an equivalent of 0.14–0.20 gamma of dried powder.

B. Caudal third-curves plotted for degree and rate of melanophore response show the same relations to changes in concentrations as described under A. The minimal concentration giving an equivalent reaction ranged from 1.4 to 3.6 gamma of dried powder.

Several authors[3,5] have reported a cluster of cells corresponding to the pars intermedia in the caudal region of the pars anterior of the chicken pituitary. Our tests show, however, that the cephalic third (the portion farthest removed from the infundibular process) of the anterior lobe of the chicken pituitary is from 10–20 times more active than the caudal third with regard to the melanophore-dispersing hormone. These results also confirm an earlier observation[3] that intermedin seems to be more concentrated in the cephalic end of the pars anterior.

* NATIONAL RESEARCH COUNCIL Fellow in the Biological Sciences.

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THE CONVEX CELLS FORMED BY SEVEN PLANES

BY H. S. WHITE

DEPARTMENT OF MATHEMATICS, VASSAR COLLEGE

Read before the Academy, April 25, 1938

1. *Introduction.*—Six intersecting planes divide projective Euclidean space into 26 convex polyedra or cells, if no more than 3 planes meet in any point. The polyedra form in this case a unique pattern or arrangement—unique, that is, if one describes them non-metrically, by numbers, contiguities and closed sequences; but when 7 planes intersect, that uniqueness is lost. The problem then arises to enumerate and describe the different sets of convex polyedra which may appear.

If we exclude situations where four or more planes pass through one point, the 7 planes have 35 points of intersection, each one a vertex of 8 cells. In every plane lie 6 lines, their segments bounding 16 convex polygons, and these all together bound and separate 42 convex polyedra, finite or infinite. There are 9 sorts of polyedra to be looked for; see figure 1. The unaided intuition is hardly equal to the problem, and *a priori* reasoning is most secure when fortified by concrete examples. Accordingly a list, believed to be exhaustive, of eleven solutions obtained by methodical use of plane diagrams may be presented here—not in full detail, of course, but in minimal essentials.

2. *Simple Convex Polyedra with 4, 5, 6 or 7 Faces.*—Not every set of integers satisfying Euler's equation can be realized by plane faces in space of 3 dimensions. Only nine meet the requirements of our present problem. These are shown by typical sketches in figure 1. To each is assigned a digit $(1, 2, 3, \ldots, 9)$; and these digits will be used to designate the several classes of polyedra.

3. *The Plane Dissected by Six Lines.*—In any one of the 7 planes the traces of the others are 6 lines dividing it into 16 convex polygons, among which are either one hexagon, or two, three or six pentagons. These four cases we may designate by *a*, *b*, *c*, *d*, respectively. In each of the 4 cases, all the lines furnish segments to the peripheries of the hexagon or pentagons:

so that when the latter are given *in situ* the mere extension of their bounding sides determines the remaining convex cells completely. But in type *b* the six lines fall into sets of 2 and 4; in type *c* into sets of 3 and 3. Therefore in constructing the system the mere numerical data do not suffice, but the species and relative shift of certain lines of intersection must be given. This remark is not relevant to the present purpose, but is important if one wishes to add details to the skeleton solutions presented here.

4. *The Problem Divided and Simplified.*—Looking at the diagrams of

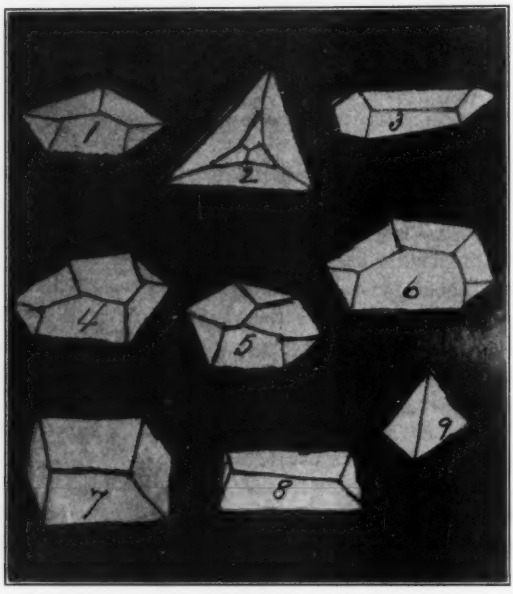

FIGURE 1
Classes of Polyedra.

figure 1, one notices that the numbers 1 to 6 have two or more faces that are pentagons or hexagons. In No. 6 there are two hexagons with a common edge or segment, in No. 3 are two pentagons united by five quadrilaterals, in each of the four others there are at least two pentagons with a common edge; in every one the hexagons and pentagons together furnish at least two lines in every plane. Given the hexagons and pentagons in relative positions, the location of quadrilaterals and triangles to complete the polyedron is seen at once; this is quickly verified by inspection.

The simplification of our problem consists now in this, that triangles and

quadrilaterals are ignored; only the faces having 5 or 6 edges are graphed schematically (see Fig. 2), with join lines showing that two faces have a common edge, or a double join line to signify the connection in type No. 3. The resulting diagram will be obviously not sufficient for the deduction of all associated polyedra of types 7, 8 and 9; that would require additional specifications which would be cumbrous in a first sketch. The diagrams do give, however, a skeleton that permits comparison and classification of all specimens, and they provide a concrete basis for more adequate description.

5. *The 11 Diagrams; Major Polyedra.*—Only one disposition of seven planes gives a structure analogous to the unique figure of six planes, namely,

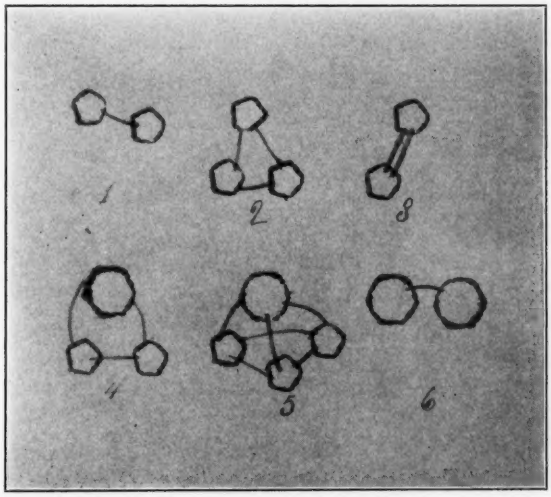

FIGURE 2
Diagrams for 6 classes of Polyedra occurring in Figures 3 and 4.

a chain of seven polyedra of type 6—a type which ought to have a suggestive name, as Bellows or Bivalve. This heptad chain, like the hexad, indicates the vicinity of a twisted cubic curve, not remote from the gauche heptagon of segments common to pairs of hexagonal faces. In all the other diagrams there occur sets of 2, 3 or 6 coplanar pentagons, and that relation is indicated on the diagram by placing the same digit in pentagons which belong in the same plane. In planes of class c or d the pentagons are actually, *in situ*, attached at certain vertices, each having one vertex in common with each other, but to place them so in the diagram would obscure the grouping.

It will be observed that polyedra of Class 1 are most numerous, forming

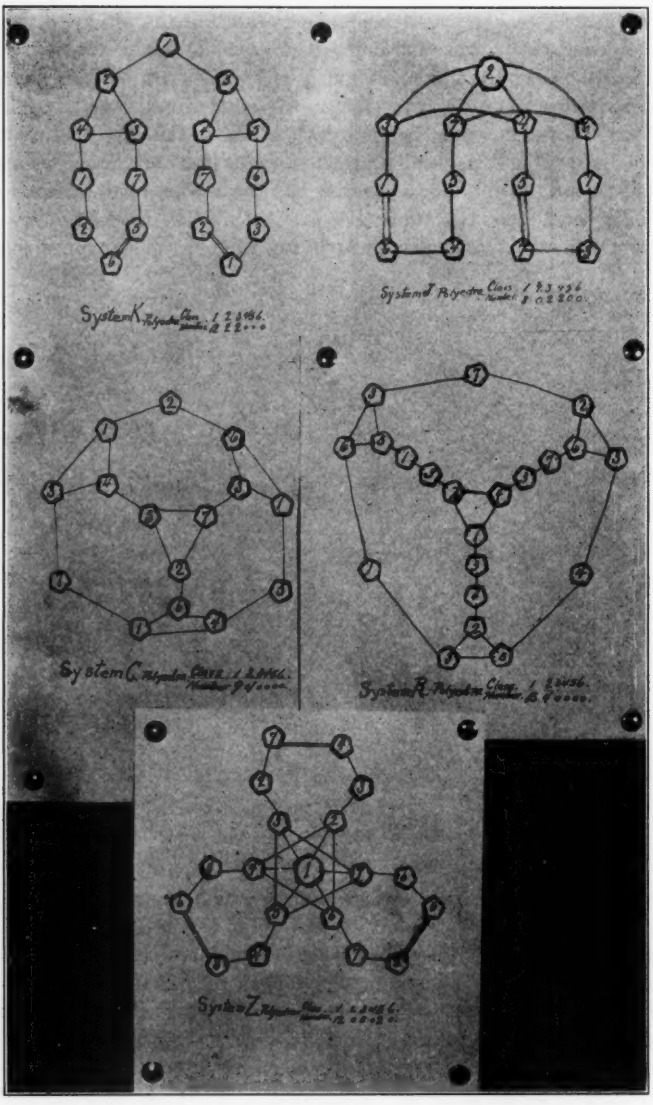

FIGURE 3

Digits designate the 7 planes. Linked polygons have common edge.

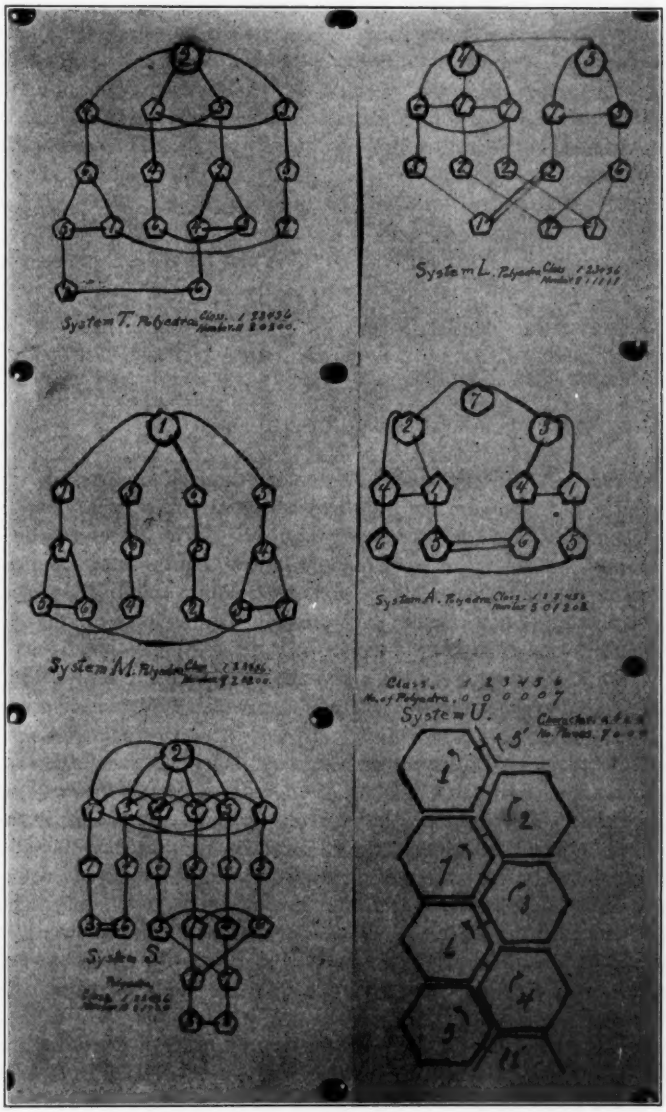

FIGURE 4

Digits designate the 7 planes. Linked polygons have one edge common.

as it were the connective tissue for those that have 7 faces. Classes 2 and 5 are geometrically most conspicuous, of course. No type is confined to a single diagram: two polyedra of Class 2, if their faces are extended indefinitely, may or may not yield the same sort of dissection of space. Beside each diagram are listed the numbers of polyedra of each class (1, 2, . . ,6), that occur in it, easily identified by the aid of figure 2. These are collected for comparison in table 1.

TABLE 1

NUMBER OF POLYEDRA OF EACH CLASS IN EACH SYSTEM

CLASS	A	L	M	S	T	U	J	K	C	R	Z
1	5	8	9	15	11	0	8	12	9	15	12
2	0	1	2	2	2	0	0	2	4	4	0
3	1	1	0	1	0	0	2	2	0	0	3
4	2	1	2	0	2	0	2	0	0	0	0
5	0	1	0	2	0	0	0	0	0	0	2
6	2	1	0	0	0	7	0	0	0	0	0

6. *Experimental Construction.*—Actual synthesis of any possible system of 7 planes may always begin from two hexagons, or two pentagons, with one edge in common. This is obvious if any cell of 6 or 7 faces is to be present, as must happen, since every plane will have a hexagon or some pentagons. One cell of Class 3 calls for no exception, for each of its pentagons must be also a face of a hexaedron of Class 1. Unless therefore the system is to contain 7 solids of Class 6 (System U of Fig. 4), two pentagons as in type 1 are taken for base, with a seventh plane whose traces in the first two are arbitrary lines meeting on the common edge but not crossing the convex pentagons.

Project the lines of both pentagons upon a plane. Use then the *Lemma: A plane containing 6 lines, of Class b, c or d, will project into a plane figure of the same class.* The same projection gives two points in the projection of each line of the 5 other planes. Locate on the plane of the drawing all such pairs of points, transfer and draw on separate sheets the lines of the 5 other planes. Now find by inspection and list by cycles of 5 or 6 digits all pentagons and hexagons in each plane. From the lists, common edges are easily discovered, also any identical cycles indicating type 3. Finally the connections are assembled in a graph, showing one of the Systems herewith presented (Figs. 3 and 4).

There are 27 possible patterns for relative situation of the initial pair of sets of 6 lines. Each one can have two of its lines so varied as to bring to coincidence and to a different separation one pair of points of intersection in a selected third plane. Hence every initial base (each of the 27) is to be looked for at least twice, barring automorphisms, which sometimes occur.

So far the survey is mainly empirical, with a minimum of theory. To

test for exhaustiveness calls for reasoning so circuitous that it is omitted here. In any case, such a conclusion requires independent checking. These diagrams are of interest, as exhibiting schematic symmetries, or rotation periods 2 or 3 or 7. I think it correct to assert further (cf. §3) that the completion of each pattern by inclusion of its minor polyedra is uniquely determinate; so that 11 appears to be the total number of systems unlike in respect of numbers and contiguities.

Not inapposite may be an *obiter dictum* from the late mathematician of wide vision, Maxime Bocher. "Moreover, although the mathematical method is the traditional one for arriving at the truth concerning geometrical facts, it is not the only one. Direct appeal to the intuition is often a short and fairly safe cut to geometric results."*

* *Conceptions and Methods of Mathematics*, Vol. 1, p. 463, of Congress of Arts and Science, Universal Exposition, St. Louis, 1904 (Boston and New York, 1905).

ON THE EXISTENCE OF MINIMAL SURFACES OF GENERAL CRITICAL TYPES

BY MARSTON MORSE AND C. TOMPKINS

INSTITUTE FOR ADVANCED STUDY AND PRINCETON UNIVERSITY

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We are concerned with minimal surfaces bounded by a simple closed curve g lying in a euclidean space of n dimensions $n > 1$. As is well known, the existence of a minimal surface of minimum type was first established by Douglas. In this connection one should refer to the significant work of Radó, McShane, Courant and others. The present paper is concerned with the existence of minimal surfaces of non-minimum as well as of minimum type. The surfaces considered are of the topological type of the circular disc. We have succeeded in making this study an application of the general theory of the critical points of functionals. That this is possible for ordinary regular positive definite problems of the calculus of variations has already been seen. Having in mind the possibility of applying the general theory to multiple integrals as well as to simple integrals, Morse has recently given an exposition [6] of the fundamental principles involved. The present paper has merely to verify the conditions laid down in the general theory. The two most difficult aspects appear in the proof of the upper-reducibility of the Douglas-Dirichlet functional and of the theorem that a homotopic critical point defines a minimal surface. Finally, an example is given in which there appear two minimal surfaces of minimum

type. By virtue of our theorems there then exists a minimal surface of non-minimum type.

The present paper is an abstract of results which will later appear in full.

1. *The General Theory.*—We shall here present only those concepts and theorems of the general theory which are essential to the theory of minimal surfaces.

We start with an abstract metric space M with points p, q, r, etc. We make use of Vietoris chains and cycles on M with coefficients in an arbitrary field. Let $F(p)$ be a real single-valued function of the point p on M. We suppose that F is finite and positive but in general not bounded. We define an *F-deformation* of M as in Morse [6]. If c is a finite constant, the subset of points of M on which $F < c$ will be said to be *below* c. The subset of points of M on which $F \leqq c$ will be said to be *below* c^+.

We shall now describe three conditions on F and M which are useful in the sequel.

I. *Bounded Compactness.*—Under this condition the subset below c^+ is compact for each finite constant c.

II. *Regularity at Infinity.*—Corresponding to each compact subset A of M, this condition requires the existence of a continuous deformation of A such that the final image of A is below some finite constant, and such that any subset of A on which F is bounded is deformed through a set of points on which F is bounded.

III. *Weak Upper-Reducibility at a Point p.*—This condition is satisfied at p if corresponding to each constant $c > F(p)$ there exists a neighborhood N_c of p relative to the set below c^+, and a deformation Δ of N_c with the following properties:

(i) For some positive η the points of N_c not below $c - \eta$ are deformed with a displacement function onto a set below $(c - \eta)^+$.

(ii) The points of N_c below $c - \eta$ remain below $c - \eta$.

Homotopic critical points and homotopic ordinary points, k-caps, a-homologies, critical sets and their type numbers are defined as in Morse [6].

The simplest general theorem is as follows:

THEOREM 1. *Let* F *be a function on* M *satisfying conditions* I, II, III *and possessing homotopic critical points which are finite in number. Let* $\mathrm{R_k}$ *be the* k*th connectivity number of* M *and* $\mathrm{M_k}$ *the sum of the* k*th type numbers of the homotopic critical points of* M. *If* $\mathrm{M_k}$ *and* $\mathrm{R_k}$ *are finite and if we set* $\mathrm{e_k} = \mathrm{M_k} - \mathrm{R_k}$ *the following relations hold*

$$\begin{aligned} e_0 &\geqq 0 \\ e_1 - e_0 &\geqq 0 \\ &\cdots\cdots\cdots \\ e_n - e_{n-1} + \cdots + (-1)^n e_0 &= 0 \end{aligned} \tag{1}$$

where n *is the maximum of the indices of the numbers* $\mathrm{M_k}$.

The condition that the homotopic critical points be finite in number will be removed in three different types of theorems. If one assumes merely that the number of homotopic critical points below each finite constant c is finite, and that the numbers M_k and R_k are finite, the relations (1) again hold with the equality deleted and n in general infinite. The least restrictive theorem of all involves the notion of a relative homology class defined as follows.

Let u be a k-cycle mod $F \leqq a$, non-bounding below c^+ mod $F \leqq a$; $c > a$. Let z be the homology class containing u taken below c^+ and mod $F \leqq a$. A constant b such that there is a cycle of z below b^+ will be termed a *cycle bound* of z. We have the following theorem:

THEOREM 2. *The cycle bounds of the relative homology class* z *have a minimum, and this minimum is assumed by* F *in at least one homotopic critical point.*

A corollary of this theorem is that the existence of at least two disconnected critical sets of minimal surfaces of minimum type implies the existence of at least one minimal surface of non-minimum type.

More generally the relations (1) can be replaced by group relations giving a decomposition of a maximal group of caps. We defer this exposition until the paper is published in full. No finiteness conditions are assumed in this general theory. Counting of type numbers is replaced by comparisons involving isomorphisms of a special type.

3. *Minimal Surface Theory.*—We shall assume that our curve g is represented in the form $x_i = g_i(s)$ where s is the arc length on g. We suppose further that $g_i(s)$ has the period 2π in s and that for each i, $g_i'(s)$ satisfies a Lipschitz condition. We shall admit other representations of g of the form

$$x_i = g_i[\varphi(\alpha)] = p_i(\alpha)$$

where $\varphi(\alpha)$ is a continuous non-decreasing function of α such that $\varphi(\alpha + 2\pi) = \varphi(\alpha) + 2\pi$. Let (r, θ) be polar coördinates in a plane of rectangular coördinates (u, v). Corresponding to the curve $[p]$ there exists a harmonic surface S of the form $x_i = x_i(r, \theta)$ defined over the disc $r \leqq 1$ and such that $x_i(1, \theta) = p_i(\theta)$. With Douglas we set

$$A(\varphi) = \frac{1}{16\pi} \int\int_Q \sum_i \frac{[p_i(\alpha) - p_i(\beta)]^2}{\sin^2 \dfrac{\alpha - \beta}{2}} \, d\alpha \, d\beta$$

where Q represents the square $0 \leqq \alpha \leqq 2\pi, 0 \leqq \beta \leqq 2\pi$.

Under the three point condition on $\varphi(\alpha)$ we understand that $\alpha = \varphi(\alpha)$ for three given distinct values of α on the interval $0 \leqq \alpha < 2\pi$. Let Ω be the space of those transformations $\varphi(\alpha)$ which satisfy the three point condi-

tion and for which $A(\varphi)$ is finite. We term φ a point on Ω. To two points φ and ψ of Ω we assign a distance

$$\varphi\psi = \max \mid \varphi(\alpha) - \psi(\alpha) \mid , \qquad 0 \leqq \alpha \leqq 2\pi.$$

As is well known the subset of points φ of Ω for which $A(\varphi)$ is at most a finite constant is compact. Thus if $M = \Omega$ and $F = A(\varphi)$ the condition I of §1 is satisfied.

We come to condition II of §1. Corresponding to each point φ of Ω we shall define a continuous deformation Δ_φ of Ω. In this deformation the time t shall run from 0 to 1 inclusive, and an arbitrary point ψ on Ω shall be replaced at the time t by the point

$$\psi^t(\alpha) = t\varphi(\alpha) + (1 - t)\psi(\alpha).$$

We introduce the integral

$$H(\psi) = \frac{1}{16\pi} \int\int_Q \left| \frac{\psi(\alpha) - \psi(\beta)}{\sin \frac{\alpha - \beta}{2}} \right|^2 d\alpha \, d\beta$$

and show that a necessary and sufficient condition that $A(\psi)$ be finite is that $H(\psi)$ be finite. It then follows readily that the space Ω is continuously deformable on itself into a point and is regular at infinity.

To establish the weak upper-reducibility of $A(\psi)$ at φ, let Q_e be the subset of Q on which $|\alpha - \beta| < e \bmod 2\pi$. We then set

$$A(\psi) = A_e(\psi) + A_e^*(\psi)$$
$$H(\psi) = H_e(\psi) + H_e^*(\psi)$$

where $A_e(\psi)$ and $H_e(\psi)$ are obtained respectively from the integrals $A(\psi)$ and $H(\psi)$ by replacing the domain of integration Q by Q_e. Referring to the deformation $\psi^t(\alpha)$ of $\psi(\alpha)$ we set

$$A(\psi^t) = a(t) \qquad A_e(\psi^t) = a(t, e) \qquad A_e^*(\psi^t) = a^*(t, e)$$
$$H(\psi^t) = h(t) \qquad H_e(\psi^t) = h(t, e) \qquad H_e^*(\psi^t) = h^*(t, e).$$

We then show that for a fixed e and bounded $A(\psi)$ the functions

$$a_i^*(t, e) \qquad a_i(t) - h_i(t, e)$$

tend to zero uniformly with respect to t as ψ tends to φ. We also need the fact that for points ψ on a set of Ω for which $A(\psi)$ is bounded the ratio of $A_e(\psi)$ to $H_e(\psi)$ tends uniformly to 1 as e tends to 0. Upon noting the effect of Δ_ϕ on $H(\psi)$ and making use of the way in which the various functionals derived from $H(\psi)$ approximate the corresponding functionals derived from $A(\psi)$, we arrive at a proof of the weak upper-reducibility of $A(\psi)$ at φ.

A transformation $\varphi(\alpha)$ of Ω which defines a minimal surface will be termed

differentially critical. We need to show that a homotopic critical point φ is a differential critical point. To that end let $D(q)$ denote the Dirichlet integral defined by the representation $q_i(\alpha)$ of g. Suppose $D(q)$ is finite and let $\lambda(\alpha)$ be a function of α of class C''' with period 2π. If e_0 is a sufficiently small positive constant and $|e| \leqq e_0$, the transformation $\mu = \alpha + e\lambda(\alpha)$ from α to μ has a single-valued inverse, $\alpha = m(\mu, e)$ and the Dirichlet integral $d(q, e)$ defined by the boundary curve

$$x_i = q_i[m(\theta, e)], \qquad |e| < e_0$$

has the form

$$d(q, e) = D(q) + eS(q) + e^2R(q, e). \tag{3.2}$$

Here $S(q)$ is an integral depending continuously on q, while $R(q, e)$ and $R_e(q, e)$ are in absolute value at most a constant multiple of $D(q)$. Moreover the condition $S(q) = 0$ implies that the harmonic surface defined by q is minimal. We can regard the transformation $\mu = \alpha + e\lambda(\alpha)$ as defining a deformation of boundary values, e being the time. Making use of (3.2), it is then easy to show that a homotopic critical point is a differential critical point. Naturally the deformations which we have used above do not lead to transformations $\psi(\alpha)$ which satisfy the three point condition, but upon making suitable conformal transformations of the disc $r \leqq 1$, the three point condition is satisfied and all is well.

All the conditions of the theory of §1 are fulfilled and the desired results about minimal surfaces can be read from the general theory.

4. *The Example.*—This example will be given in the space of coördinates x, y, z. Consider the closed curve composed of the following four arcs taken successively:

$$\begin{array}{llll} x = e & (z-a)^2 + y^2 = 1 & z \geqq 0 \\ y = e & (z+a)^2 + x^2 = 1 & z \leqq 0 \\ x = -e & (z-a)^2 + y^2 = 1 & z \geqq 0 \\ y = -e & (z+a)^2 + y^2 = 1 & z \leqq 0, \end{array}$$

where e and a are positive constants with $a^2 = 1 - e^2$. Let C_e be a closed curve obtained by rounding off the corners of the above closed curve. If e is sufficiently small and the rounding off process is suitably made, it is easy to see that there exist two minimal surfaces of minimum type bounded by $C(e)$. Moreover, these two surfaces do not define points of Ω in the same connected critical set. It follows from our theory that $C(e)$ bounds at least one minimal surface of non-minimum type.

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ADDITIVE SET FUNCTIONS ON GROUPS

BY S. BOCHNER

DEPARTMENT OF MATHEMATICS, PRINCETON UNIVERSITY

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1. We consider on an arbitrary set G a family X of subsets $\{E\}$ with the properties

1) X contains the empty set 0; if $E \epsilon X$, then $G - E \epsilon X$
2) if $E_1, E_2 \epsilon X$, then $E_1 \cdot E_2 \epsilon X$ and $E_1 + E_2 \epsilon X$,

and a Jordan volume vE with the properties

3) $0 \leq vE \leq 1, \quad v0 = 0, \quad vG = 1$
4) if $E_1 \cdot E_2 = 0$, then $v(E_1 + E_2) = vE_1 + vE_2$.

We do not assume that X is completely additive, or that vE is completely additive relative to X. The class of point functions $f(x)$ which are Riemann integrable relative to vE will be denoted by R, the integral of $f(x)$ over any set E will be denoted by $\int_E f(x)dv$ and the (incomplete) Banach space of functions $f(x)$ with the norm

$$||f||_p = \left(\int_G |f(x)|^p dv\right)^{1/p} \text{ will be denoted by } R_p,\ p \geq 1.$$

2. Many properties of (finitely) additive set functions of bounded variation which are usually proven under assumptions of complete additivity can be established for our general case. It can be shown that in the Banach space AC of absolutely continuous set functions $F(E)$ with the variation as norm, the finitely valued functions are dense. In other words, the space AC is not greater than, but equal to, the Banach closure of the space R_1. The same is true of the Banach spaces V_p, $p > 1$, of the set functions $F(E)$ with the norm

$$||F||_p = \sup \left(\sum_{n=1}^{N} |F(E_N)|^p \cdot |vE|^{1-p}\right)^{1/p},$$

$E_1+\ldots+E_N = G$. In other words V_p is equal to the closure of R_p, $p > 1$. These theorems are a generalization of the theory of O. Nikodym.[1]

3. Another peculiar feature is the possibility of constructing the Jordan volume from the Riemann integral. The precise statement is as follows:

Let C be a class of bounded functons $\{f(x)\}$ on G and let $M_x f(x)$ be a number which is defined for all $f(x)$, and let the following properties hold. 1) C contains the function $f(x) = 1$, 2) if $f_1, f_2 \epsilon C$ and c_1, c_2 are numbers, then $f_1 f_2 \epsilon C$, $c_1 f_1 + c_2 f_2 \epsilon C$, 3) if $f \epsilon C$, then $\bar{f} \epsilon C$, and if f is real, then $|f| \epsilon C$, 4) $M_x 1 = 1$, 5) $M_x(c_1 f_1 + c_2 f_2) = c_1 M_x f_1 + c_2 M_x f_2$ and 6) $M_x f(x) \geq 0$ if $f(x) \geq 0$.

Then there exists a Jordan field X and a volume vE, such that C belongs to R and

$$M_x f(x) = \int_G f(x) dv.$$

Also the class C is dense in each R_p, $p \geq 1$.[2]

4. If G is a group, and C consists of almost periodic functions on it,[3] it is appropriate to assume that C is group invariant, that is if $f(x) \epsilon C$ and $a \epsilon G$, then $f(x^{-1}), f(xa), f(ax) \epsilon C$. The generated field X and the volume vE are also group invariant. In this case if $\{\varphi_\alpha(x)\}$ is an orthogonal system in C consisting of representation coefficients and if $\{\varphi_\alpha(x)\}$ is complete in C it is also complete in AC. Thus any set function $F(E) \epsilon AC$ is uniquely determined by its Fourier expansion

$$(*) \qquad \sum_\alpha C_\alpha \varphi_\alpha(x), \qquad C_\alpha = \int_G \overline{\varphi_\alpha(x)} dF,$$

the latter integral being a Riemann-Stieltjes integral of the usual kind.

In particular, a series (*) is the Fourier series of a function $F(E)$ from V_2, if and only if $\sum |C_\alpha|^2 < \infty$ (*Riesz-Fischer theorem*). This is a partial generalization of the theory of A. S. Besicovitch to almost periodic function on general groups.[4]

5. We finally consider the space V_C of set functions $F(E)$ with the norm

$$||F|| = \sup_{f \epsilon C} \left| \int_G f(x) dF \right|.$$

This leads to the following *uniqueness theorem*.

If an element $F(E)$ from V_C is right invariant, that is if $F(Ea)$ is independent of a (or if it is left invariant), then there exists a constant λ such that $F(E) = \lambda \cdot vE$ for $E \epsilon X$.

Furthermore, calling a function $F(E)$ from V_C absolutely continuous if it is the limit in norm of functions $\Phi(E)$ which are indefinite integrals of point functions, that is

$$\Phi(E) = \int_E \varphi(x) dv,$$

then the following theorem holds. A function $F(E) \epsilon V_c$ is absolutely continuous if the function $F(Ea)$ from G to V_c is almost periodic.[5] This generalizes a theorem of A. Plessner for functions in one variable.[6]

6. These theorems have applications to functions of one variable which will be presented in another publication.

[1] "Sur une generalisation des integrales de M. J. Radon," *Fundam. Math.*, **15**, 131–179 (1930).

[2] This theorem was suggested by J. v. Neumann's construction of the Haar measure on compact groups, in *Compositio Mathematica*, **1**, 106–114 (1934).

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